

Division Tricks

Division trick for number 9

This trick determines when a number is divisible by 9.

Example 1: Consider 9,432. We first take the sum of the digits

$$9 + 4 + 3 + 2 = 18.$$

Then we take the sum of the digits of the sum:

$$1 + 8 = 9.$$

If this number is divisible by 9, then the original number is divisible by 9.

Example 2: Is the number 376 divisible by 9?

$$3 + 7 + 6 = 16$$

$$1 + 6 = 7$$

This means that when you divide this number by 9 you will have a remainder of 7. Hence, the number **IS NOT** divisible by 9!

Division trick for number 3

You can use a trick for number 9 to determine if the number is divisible by 3.

Example 1: Is the number 124,573 divisible by 3?

$$1 + 2 + 4 + 5 + 7 + 3 = 22$$

$$2 + 2 = 4$$

Now we divide 4 by 3 and we have a remainder of 1. This means that when you divide the original number, 124573, by 3 you will have a remainder of 1. Hence, the number 124,573 **IS NOT** divisible by 3.

Division trick for number 11

This trick determines when a number is divisible by 11.

Example 1: Say you want to divide 15,345 by 11.

Trick: Start by writing a “+” above the most right digit you wish to divide by 11. So in our case, we write “+” above the digit 5. Then, write a “-” above the second right most digit you wish to divide by 11. So for our example, we write a “-” above the digit 4. So if we continue this osculating process we proceed to place “+” above the digits 3 and 1 and we put a “-” above the 5. Finally, we have,

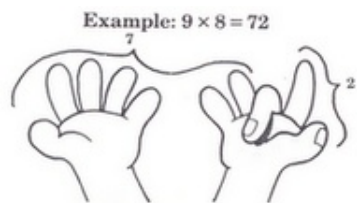
$$\begin{array}{cccccc} + & - & + & - & + & \\ 1 & 5 & 3 & 4 & 5 & \end{array}$$

Now, we group our “+” and “-“ together, add them together, and then subtract the two. As shown below,

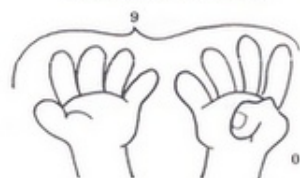
$$(5 + 3 + 1) - (5 + 4) = (9) - (9) = 0$$

Hence, our result is 0. This indicates that our proposed number **IS** divisible by 11. Furthermore, if your remainder is not 0 or 11, where k is any integer, then the number has a remainder when you divide by 11.

How do you calculate multiplication of 9 on your hands?



Bend over finger #8
Example: $9 \times 10 = 90$



Bend over finger #10
9 fingers on the left and 0 fingers on the right = 90.

For 9×1 you put down your first finger and count the ones left.

Hence, $9 \times 1 = 9$.

For 9×2 you put down your second finger and count the fingers to the left and right. On the left there is 1 and on the right there are 8.

Hence, $9 \times 2 = 18$.

For 9×3 you repeat the steps as before. This time you will have 2 fingers of the left of your finger that is down and 7 to the right.

Hence, $9 \times 3 = 27$.

Further Study: Division Tricks

Magic of symmetry

Calculate using a calculator. Look at the magic of symmetry!

$$1 \times 8 + 1 =$$

$$12 \times 8 + 2 =$$

$$123 \times 8 + 3 =$$

$$1234 \times 8 + 4 =$$

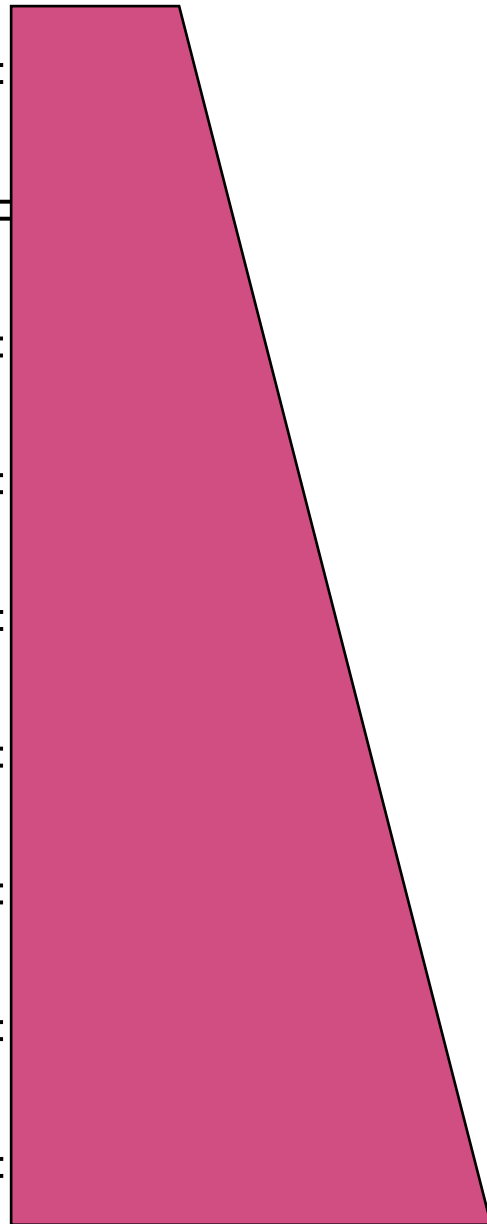
$$12345 \times 8 + 5 =$$

$$123456 \times 8 + 6 =$$

$$1234567 \times 8 + 7 =$$

$$12345678 \times 8 + 8 =$$

$$123456789 \times 8 + 9 =$$



Divisibility problems

- Use the trick of 3 and determine if 55550671 is divisible by 3.
- Use the trick of 9 and determine if 345609 is divisible by 9.
- Use the trick of 9 and determine if 3421829 is divisible by 9.
If it is not find the remainder.
- Use the trick of 11 and determine if 5238167 is divisible by 11.
- If $359x4$ is divisible by 9, compute x . (Answer: 6)
- If $35x94$ is divisible by 3, How many values does x have? (Answer: 4)
- If $3x567$ is divisible by 11, compute x (Answer: 9)
- If $4a476b2$ is divisible by 11, compute $a + b = ?$ (Answer: 9)
- If $5a476b1$ is divisible by 11, how many values does a have? (Answer: 10)
- Search, if there exists division tricks for 8 and 13.

References

1. Briggs C.C., *Simple divisibility rules for the 1st 1000 prime numbers.*
<http://arxiv.org/ftp/math/papers/0001/0001012.pdf>
2. Kolpas, S.J., *Let your fingers do the multiplying*
http://www.dccc.edu/sites/default/files/faculty/sid_kolpas/mathteacherfingers.pdf
3. Mathemagics – Mental Math Tricks.
<https://itunes.apple.com/us/app/mathemagics-mental-math-tricks/id306586847?mt=8>
4. Common Core State Standards for Mathematics.
http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Common Core: Division Tricks

Objectives:

- To learn the math behind the divisibility tricks, e.g., some divisibility rules come from modulus of the number.
- To define more general algorithms - multiplying by fingers can be expanded to more general algorithms and the students they can practice by counting on their fingers – for example the finger trick for 9 is based on a theorem from number theory that states that if number is a multiple of 9, then the sum of its digits is a multiple of nine. Discovering general algorithm would be a challenging lesson for students.
- Motivate students to look up and learn other math tricks.
- Developing skills - become a math \square magician: for example, the student can amaze friends and family by learning divisibility rules which provides a student with the skill to see if something is divisible at lightning fast speed.
- To just have FUN - learning and practicing the tricks of math calculation in a fun and engaging way enhances student learning.

Common Core Standards:

The divisibility tricks and learning finger multiplication techniques tricks fall in the context of the following standards for mathematical practice:

1. Reason abstractly and quantitatively
2. Look for and make use of structure