Title: Financing Superheroes

## Brief Overview:

Students will look at the financial needs of Superheroes in both investments and loans to finance their crime fighting activities. The students will begin with the use of logs, natural logarithms and calculator intersections of graphs to solve varied compound interest problems. Next, the students will be introduced to loan calculations using the Financial Applications of the TI-84 and determine the best financial choices. Finally, the students will create a Superhero credit card comic strip to fight the crime of money mismanagement.

## NCTM Content Standard/National Science Education Standard:

NCTM Principles for Mathematics

- Equity. Excellence in mathematics education requires equity - high expectations and strong support for all students
- Curriculum. A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.
- Teaching. Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
- Learning. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
- Assessment. Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.
- Technology. Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

NCTM Data Analysis and Probability Standards for Grades 9 - 12

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- Select and use appropriate statistical methods to analyze data
- Develop and evaluate inferences and predictions that are based on data


## Grade/Level:

High School Pre-Calculus

## Duration/Length:

Three 60-minutes lessons

## Student Outcomes:

Students will be able to:

- calculate and compare Principal, Rate and Interest problems with varied compounding times per year along with continuous compounding.
- confirm logarithmic and/or algebraic solutions to Compounding Interest problems by solving graphically.
- be introduced to the Financial Applications of the TI-84 calculator to determine desired loan financial information.
- compare financial choices in compounding and loan financial problems and make appropriate decisions.


## Materials and Resources:

- TI - 83/84 graphic calculator and overhead panel
- Overhead transparency of Homework Charts, if desired.
- Prepared "Who has - Superhero does" class activity cards - 3x5 cards in 4 colors
- Teacher Notes for Lesson 2
- "Money When?" Station set up, printed worksheets and answer sheets
- Copies of worksheets
o Compound What
o Finance Where


## Development/Procedures:

Lesson 1
Pre-assessment - Student have already studied both common and natural logarithms. The students have solved exponential and logarithmic equations. Students will participate in the "Who has - Superhero does" activity to review logarithms. Divide the class into groups of six students. Refer to the "Who has - Superhero does" Teacher Instruction sheet. Note that this activity involves cards which the teacher must prepare prior to the lesson.

Launch - Students will be given worksheet "Compound What?" which asks students to find out which bank account would be most beneficial for the Super Hero

Teacher Facilitation - Work through worksheet "Compound What?" to explain the different types of compounding interest. Examples will be presented that will have students solve for various variables in the interest equations.

Student Application - Students will complete the chart at the end of the "Compound What?" worksheet to practice solving for the various variables presented in class.

Embedded Assessment - Students will be observed by the teacher as they work on the chart.

Re-teaching/Extension - Students will have to complete the chart that was begun in class. Homework will be reviewed the following day and any problems will be addressed. Students will be encouraged to discuss finances with parents concerning investments.

## Lesson 2 <br> Pre-assessment- Review with students the completion of Lesson 1

 "Compound What?" chart solutions.Launch - Use two problems from the "Compound What?" chart to confirm solutions through graphic method. Using the TI84 graphing calculator have students put problems \# 5 into the calculator to solve using the method of intersecting lines. The students should make the missing time be represented by the variable $x$. Have students confirm logarithmic results determined for question \#5.

Example:
\#5 $\quad \mathrm{y}_{1}=50000\left(1+\frac{.09}{12}\right)^{12 x}$

$$
\mathrm{y}_{2}=100000
$$



## Example:

Have students keep investment \# 5 as $\mathrm{y}_{1}$ and also enter the investment problem \# 6 as y 2 . Classroom discussion of the graphs should include which is the best and discussion of additional choices of monthly, daily, quarterly compounding. Alter \# 5 to quickly analyze the changes in the investment compared to continuous.

Through classroom discussion from the launch, the teacher initiates further financial discussion to include financing loans instead of compound interest.

Teacher Facilitation - Continue to lead the class discussion to the introduction of the Finance Applications of the TI84 for calculating loan payments. Refer to the "Teacher Notes for Lesson 2" for a detailed explanation on how to use the finance applications, as well as provided student examples. Students are given worksheet "Finance Where?" to discover the how to use the TI84 Finance Application to compare loan options.

Student Application - Complete the "Finance Where?" worksheet. Results will be presented in the final 10 minutes of class with justification for choice of loan.

Embedded Assessment - Students will be observed by the teacher as they complete the "Finance Where?" worksheet. In addition, through verbal presentation the teacher will be able to evaluate student understanding of loan choice.

Re-teaching/Extension - Student expectation is to complete the "Finance Where?" worksheet during the class period. If not completed, student would need to complete for homework. In addition, students should bring in real world financial rates from financial institutes for various types of loans.

Lesson 3 Pre-assessment - Class discussion of loan advertisements brought in by students.

Launch - Further class discussion of different types of loans including credit cards, retirement, car loans, etc.

Teacher Facilitation - Divide the class into groups of three students per group. Students will complete the four assigned stations with problems presented on worksheets.

Student Application - Students will complete the four Superhero stations on provided answer form

Embedded Assessment - "Money When?" answer sheet from stations will be used as exit tickets.

Re-teaching/Extension -Using the loan advertisements brought in by students, students could repeat the processes presented with real life examples.

## Summative Assessment:

Students will create a Superhero comic strip. This final assessment allows evaluation of student math understanding of Finances while including writing and cross-curricular subjects of art, computer graphics and design.

Superhero Comic Assignment Instructions: Assign the students to make themselves into a Superhero trying to stop the crime of money mismanagement. They should create a comic strip that will rescue a victim from the Credit Card repayment of $\$ 500$ at $21 \%$ interest. Credit Cards are compounded and paid monthly. Instructions should include in the comic the amount of time and the total amount the victim would have to repay the Credit Card Company if he/she was only making the minimum monthly payment of $\$ 35$. Encourage the students to be creative. They should use one piece of $8 \times 10$ Paper with a minimum of 4 panels. The student could sketch, use computer graphics, cut and paste, etc.

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Who has - Superhero does
Teacher Instructions

## Preparation for Activity:

1) Copy a set of each round of the "Who has - Superhero does" playing cards for each group of six students in the classroom. There are four rounds of cards.
2) Cut out each set of the "Who has - Superhero does" playing cards. Paste the Round One cards onto blue index cards, paste the Round Two cards onto yellow index cards, paste the Round Three cards onto white index cards, and paste the Round Four cards onto orange index cards. (It is suggested that the cards be laminated or placed in baseball card holder to protect for future classroom use.)
3) Place each set of cards in separate envelopes, so that each group of six students will receive one set of each round of cards. Each group of students will play all four rounds of the "Who has - Superhero does" activity.

## Activity Direction:

1) Divide the classroom into groups of six students
2) Distribute envelopes containing the four rounds of colored sets of index cards to each group so that each group receives one set of each round of cards.
3) Give the following instructions for the class activity:

- Each group will play the "Who has -Superhero does" game in four rounds.
- Each round is as follows:
o The first round is to review common log simplification.
o The second round is natural log simplification.
o The third and fourth rounds are common and natural log simplification of expressions.
- Each group has six cards of four different colored index cards representing the four rounds.
- Begin with Round One, which are the blue index cards Distribute one blue card to each person in the group.
- Instruct students not to discuss what is on their card, or show it to anyone in the group.
- Each card has two statements - one statement with "Superhero does" and a second statement with "Who has". It does not matter who begins the game but the first person only begins with the "Who has" statement.
- The first student reads their "Who has" out loud to the group. After mental calculation, the student holding the card that has the correct solution responds out loud "Superhero does" and gives the solution. Students should explain to the group which method or laws used to determine the solution. Then, this student reads their "Who has" part, and the round continues until the last person with the "Superhero does" was the student who first began the round.


## "Who has - Superhero does" Playing Cards

 (Log/ Ln Review Rounds One and Two)| $\begin{gathered} \text { Round } 1 \\ \text { Common Logs } \\ \hline \end{gathered}$ | Round 2 Natural Logs |
| :---: | :---: |
| Batman does $\frac{1}{2}$ <br> $1 / 2$ because Batman is half of a team with Robin <br> Who has $\log 10 \quad\left(\right.$ or $\left.\log _{2} 2\right)$ | The Flash does $\frac{1}{2}$ <br> Who has lne |
| Superman does 1 <br> because Superman always works alone! <br> Who has $\quad \log 0 \quad\left(\right.$ or $\left.\log _{5} 0\right)$ | Ghost Rider does 1 <br> Who has $\ln 0$ |
| Bat Woman does <br> $\varnothing$ or no solution Does she even exist? <br> Who has $\quad \log \frac{1}{10} \quad\left(\right.$ or $\log _{2} \frac{1}{2}$ ) | Wonder Woman does $\varnothing$ or no solution Who has $\ln \frac{1}{e}$ |
| Spiderman does -1 <br> Because Spiderrman can defy gravity and move in a negative direction with ease <br> Who has $\log 1 \quad\left(\right.$ or $\left.\log _{2} 1\right)$ | Super Girl does -1 <br> Who has $\ln 1$ |
| Iceman does 0 <br> Because Iceman operates at freezing temperature <br> Who has $\log 10000 \quad\left(\right.$ or $\left.\log _{2} 16\right)$ | Brainiac V does 0 <br> Who has $\ln e^{4}$ |
| The Fantastic Four do 4 Can you name them? <br> Who has $\quad \log \sqrt{10} \quad$ (or $\log _{2} \sqrt{2}$ ) | Cyclops does 4 <br> Who has $\ln \sqrt{e}$ |

## "Who has - Superhero does"

(Log/Ln Expressions Review Rounds Three and Four)

| Round 3 Common Log Expressions | Round 4 <br> Ln Expressions |
| :---: | :---: |
| Storm does $\frac{\log 5}{\log 3}$ or $\log _{3} 5$ <br> Who has $\log _{2} 8+\log _{2} 2$ | Captain America does $\frac{\ln 5}{\ln 3}$ <br> Who has $\ln e+\ln e^{2}$ |
| Rogue does 4 <br> Who has the value of $x$ when $\log x=\frac{1}{2}$ | Wolverine does 3 <br> Who has the value of $x$ when $\ln x=7$ |
| Robin does $\sqrt{10}$ <br> Who has the value of $x$ when $2^{x}=32$ | Green Lantern does $e^{7}$ <br> Who has the value of $x$ when $e^{x}=5$ |
| The Atom does 5 <br> Who has the value of $x$ when $10^{x+3}=1000$ | Iron Man does $\ln 5$ <br> Who has the value of $x$ when $e^{x-2}=5$ |
| The Torch does 0 <br> Who has the value of $x$ when $\log x+\log x=2$ | Invisible Girl does $2+\ln 5$ <br> Who has the value of $x$ when $\ln x^{2}-\ln x=0$ |
| Hulk does 10 <br> Who has the value of $x$ when $3^{x}=5$ | Green Arrow does 1 <br> Who has the value of $x$ when $3^{x}=5$ |

$\qquad$
Date: $\qquad$
Robin goes to the Gotham Bank to open up a savings account with the money he has earned working at the Justice League of American as an intern for the summer. The teller explains to him that there are three types of savings accounts that he could open. The first account earns simple interest; the second account earns compound interest; while the third account is compounded
 continuously.

Robin tells the bank teller that he has $\$ 1500$ to put in the savings account and he will not touch the money for 5 years. The bank teller informs Robin that at the Gotham Bank the interest rate for savings accounts is $6 \%$ and the bank compounds quarterly. Robin wants to find out which type of account would earn him the most money.

The bank teller gives him three formulas so that Robin can compare the three types of accounts

| Simple Interest | Compound Interest | Continuous Compounding |
| :---: | :---: | :---: |
| $A=P+P r t$ | $A=P\left(1+\frac{r}{n}\right)^{n t}$ | $A=P e^{r t}$ |

1) Identify, in words, what the variables represent in each formula.

| Variable | Represents |
| :---: | :---: |
| $A$ |  |
| $P$ |  |
| $r$ |  |
| $t$ |  |
| $n$ |  |

2) Identify the various values of $n$ based on when money is compounded.

| Compounded | Value of $n$ |
| :---: | :---: |
| Annually |  |
| Quarterly |  |
| Monthly |  |
| Daily |  |

3) Calculate the amount of money Robin would have in each account after 5 years. Show all your work that leads to your answers.

| Simple Interest | Compound Interest | Continuous <br> Compounding |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |

4) Which type of account should Robin open?
5) What would be the difference in the value of the account if the bank compounded daily instead of quarterly?


Use the compound interest formulas to answer the questions below. Show work for each problem.

1) The Thing saved money in building his home because he could create the rock himself. He invested the extra $\$ 25,000$ in an account that was earning $7.5 \%$ interest compounded monthly. Find the amount in The Thing's account after 10 years.
2) Mr. X currently has a slush fund to repair the damages done during training exercises at School for Gifted Children located at Mutant headquarters. Mr. X has $\$ 17,310.98$ in an account that was earning $5 \%$ interest compounded daily. How much money did Mr . X start with 10 years ago?
3) Green Arrow stayed on a straight and narrow path to save money. He has a balance in his savings account of $\$ 32,000$. He indicated that 20 years ago he had invested $\$ 12,000$ in an account that was compounded quarterly. What interest rate was Green Arrow earning?
4) Batman's wallet was too thick to hold all of his money so he deposited $\$ 75,000$ in an account that was earning $53 / 4 \%$ interest compounded daily. How many years would it take for Batman to double his money?

Compound What?
Name $\qquad$
Date: $\qquad$

Complete the chart below. Show all work in the space provided below the table.

| Problem | Amount in <br> account | Principle | Interest <br> Rate | \# of times <br> Compounded | Time <br> (years) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ |  | $\$ 5,600.00$ | $5 \frac{1}{4} \%$ | Monthly | 6 |
| $\mathbf{2}$ | $\$ 15,800.00$ |  |  |  |  |
| $\mathbf{3}$ | $\$ 7,500.00$ | $\$ 5,765.00$ |  | Daily | 10 |
| $\mathbf{4}$ | $\$ 60,000.00$ | $\$ 42,000.00$ | $8 \%$ | Annually |  |
| $\mathbf{5}$ | $\$ \$ 00,000.00$ | $\$ 50,000.00$ | $7 \%$ | Continuous |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Use the table below to show work, justifying your answers to the previous table.

| \#1 | \#2 |
| :--- | :--- |
|  |  |

$\qquad$
Date: $\qquad$
Robin goes to the Gotham Bank to open up a savings account with the money he has earned working at the Justice League of American as an intern for the summer. The teller explains to him that there are three types of savings accounts that he could open. The first account earns simple interest; the second account earns compound interest; while the third account is compounded
 continuously.

Robin tells the bank teller that he has $\$ 1500$ to put in the savings account and he will not touch the money for 5 years. The bank teller informs Robin that at the Gotham Bank the interest rate for savings accounts is $6 \%$ and the bank compounds quarterly. Robin wants to find out which type of account would earn him the most money.

The bank teller gives him three formulas so that Robin can compare the three types of accounts

| Simple Interest | Compound Interest | Continuous Compounding |
| :---: | :---: | :---: |
| $A=P+P r t$ | $A=P\left(1+\frac{r}{n}\right)^{n t}$ | $A=P e^{r t}$ |

1) Identify, in words, what the variables represent in each formula.

| Variable | Represents |
| :---: | :---: |
| $A$ | Amount in account |
| $P$ | amount of money invested |
| $r$ | interest rate as a decimal |
| $t$ | time in years |
| $n$ | number of times compounded |

2) Identify the various values of $n$ based on when money is compounded.

| Compounded | Value of $n$ |
| :---: | :---: |
| Annually | 1 |
| Quarterly | 4 |
| Monthly | 12 |
| Daily | 365 |

3) Calculate the amount of money Robin would have in each account after 5 years. Show all your work that leads to your answers.

| Simple Interest | Compound Interest | Continuous <br> Compounding |
| :---: | :---: | :---: |
| $\$ \mathbf{1 9 5 0}$ | $\$ 2020.28$ | $\mathbf{\$ 2 0 2 4 . 7 9}$ |

4) Which type of account should Robin open?

## Continuous compounding

5) What would be the difference in the value of the account if the bank compounded daily instead of quarterly?
$\$ 2024.74-\$ 2020.28=\$ 4.46$


Use the compound interest formulas to answer the questions below. Show work for each problem.

1) The Thing saved money in building his home because he could create the rock himself. He invested the extra $\$ 25,000$ in an account that was earning $7.5 \%$ interest compounded monthly. Find the amount in The Thing's account after 10 years.
$\$ 52801.62$
2) Mr. X currently has a slush fund to repair the damages done during training exercises at School for Gifted Children located at Mutant headquarters. Mr. X has $\$ 17,310.98$ in an account that was earning $5 \%$ interest compounded daily. How much money did Mr. X start with 10 years ago?
$\$ 10500.00$
3) Green Arrow stayed on a straight and narrow path to save money. He has a balance in his savings account of $\$ 32,000$. He indicated that 20 years ago he had invested $\$ 12,000$ in an account that was compounded quarterly. What interest rate was Green Arrow earning?

## 4.9\%

4) Batman's wallet was too thick to hold all of his money so he deposited $\$ 75,000$ in an account that was earning $53 / 4 \%$ interest compounded daily. How many years would it take for Batman to double his money?

## 12.1 years

## Compound What?

Name $\qquad$
Date: $\qquad$

Complete the chart below. Show all work in the space provided below the table.

| Problem | Amount in account | Principle | Interest <br> Rate | \# of times Compounded | Time (years) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | \$7668.18 | \$5,600.00 | $51 / 4 \%$ | Monthly | 6 |
| 2 | \$15,800.00 | \$7463.97 | $7112 \%$ | Daily | 10 |
| 3 | \$7,500.00 | \$5,765.00 | 3.3\% | Quarterly | 8 |
| 4 | \$60,000.00 | \$42,000.00 | 8\% | Annually | 4.63 |
| 5 | \$100,000.00 | \$50,000.00 | 9\% | Monthly | 7.73 |
| 6 | \$100,000.00 | \$50,000.00 | 7\% | Continuous | 9.9 |

Teacher Notes for Lesson 2
A) Teacher should lead the classroom discussion from comparisons of investments to discussion of loans choices.

## Discussion Questions from teacher to students:

1) What other types of interest are you familiar with?

- Loan, home loans or mortgage loans, credit cards, student loans, car loans, etc.

2) How do you think the formula for Compound Interest that you earn could also be used to calculate the interest you would pay for a simple loan?

- Example problem:
a) savings account $\$ 1000$ at $3 \%$ interest rate for one year yields $\$ 1030$ in the account or $\$ 30$ in interest
b) a loan of $\$ 1000$ where you are paying $3 \%$ rate for one year you would owe $\$ 1030$ which is $\$ 30$ in interest paid to the loan facility

3) How much would my monthly payments be on either a house or a car if I knew the purchase price?

- Let us look at some loan calculations using your TI84.
B) Students will then need TI83/84 calculator. Introduce the Finance Applications. Note to teacher: Refer to the TI 84 Manual TMV Solver page 93 and/or page 166 for additional vocabulary and explanation. Use the same instructions for both TI 83Plus/TI 84 calculators.
C) Teacher needs to review the following vocabulary as part of a loan using the examples Basic Loans. In addition, there are provided examples including the screen entries to calculate the number of payments to pay off a loan with various changes. See possibilities in examples \# 1-3:

Please be aware that Begin and End choice will sometimes give a different result. The "begin" and "end" setting is the time within the period for payment. Both should be demonstrated to students for clarification for example: If payment is due at the beginning of the month then choose "begin".
D) Begin with the TI84 calculator demonstrated with overhead view screen

Press Apps, Finance ( 1), then select 1 TVM Solver.

## Using TVM Solver Vocabulary:

| $\mathrm{N}=$ | Number of payments |
| :---: | :---: |
|  | (In example \#1 monthly would be 12 payments, see example \#2 for other choices.) |
| $\mathrm{I} \%=$ | Interest Rate |
|  | (In example \#1 interest rate would be 5 - do not convert to decimal or use \% sign) |
| $\mathrm{PV}=$ | Present Value (or Loan Amount) |
|  | (In example Present Value or Loan Amount is 1000- do not use the \$ sign or decimals) |
| PMT $=$ | Payment Amount |
|  | (in example \#1 of payment amount as - 50 because you are paying |
|  | $\$ 50$ each month and reducing the amount owed. Note: this is always a negative value, be sure to use negative sign and not subtraction key) |
| $F V=$ | Final Value of Loan |
|  | (in example \#1 the final value would be zero because you want to pay off the entire loan. It does not always have to be zero. See example \#3) |
| $\mathrm{P} / \mathrm{Y}=$ | Number of payments per year |
|  | (in example \#1 is monthly so number of payments is 12) |
| $\mathrm{C} / \mathrm{Y}=$ | Number of times loan is compounded per year |
|  | (in example \#1 the loan was compounded monthly therefore use 12) |

## Examples of a Basic Loan: \# 1-3

Begin with basic loan of $\$ 1000$ at $5 \%$ and compounded monthly, demonstrate the following for the students on overhead while students enter on their calculators.

Note: The same ideas could be used for mortgage, car or student loans of differing values.

Calculator Note: Calculator challenge: enter any value in N and then enter the data in the other fields for I, PV, PMT, FV P/Y and C/Y. Then return to N and delete for empty field. Then solve by input solve (key strokes of alpha/enter). Answer will be in the field desired. This is true for all calculations.

## Basic Loan Examples (using "end" setting):

1) If we make monthly payments of $\$ 50$, how many payments would we have to make?

| Calculator Set Up | Answer input solve (alpha/enter) in $\mathrm{N}=$ |
| :---: | :---: |
|  |  |

2) What would be the payment if we paid off the loan in 2 years?

| Calculator Set Up | Answer input solve (alpha/enter) in $\mathrm{N}=$ |
| :---: | :---: |
|  |  |

3) How long making payments of $\$ 100$ to pay off half of the loan?

| Calculator Set Up | Answer input solve (alpha/enter) in $\mathrm{N}=$ |
| :---: | :---: |
| $\begin{aligned} & \mathrm{N}=\square \\ & \mathrm{T}=5 \\ & \mathrm{P}=1060 \\ & \mathrm{P}=10=100 \\ & \mathrm{P}=100 \\ & \mathrm{~V}=12 \end{aligned}$ <br> FMT:ENL BEGIN |  |

E) Next teachers should lead the students into other loan questions.

Discussion Question: "Do you think there are other questions about loans beyond just how long to pay off a loan and calculating monthly payments?

- Yes how much would I actually pay for a car or a house with the principal and interest (for mortgage see examples "b- d"


## Example of a Mortgage (using TMV Solver in same manner):

Begin with basic mortgage of $\$ 200000$ at $8 \%$ interest for 30 years. Assume compounded and payments made monthly.
a) Determine the monthly payment with loan amount known

| Calculator Set Up | Answer input solve (alpha/enter) in $\mathrm{N}=$ |
| :---: | :---: |
| $\begin{aligned} & \mathrm{N}=360 \\ & \mathrm{P}=8 \\ & \mathrm{PM}=20010 \\ & \mathrm{PN}=0 \\ & \mathrm{PV}=12 \\ & \mathrm{FH}=12 \\ & \text { FMTE BEGIH } \end{aligned}$ |  |

## Continued class discussion:

To determine respective aspects of the loan, follow the directions below:

## Calculator instructions:

TMV Solver must still have the data input for the loan. Continue with the same loan in example "a" for a mortgage.

1) Clear Screen
2) Key stroke APPS, choose Finance (1), Choose one of the following:

- $\quad \sum \operatorname{Pr} n(\quad$ which is choice ( 0 ) to determine the Principal paid. See example "b"
- $\quad \sum \operatorname{Int} \quad$ which is choice (A) to determine the Interest paid.

See example "c"

- Bal ( which is choice (9) to determine the balance of the mortgage. See example "d"


## Continued Mortgage Examples:

b) To determine the total amount of Principal paid in the first 3 years

Example: Apps $\rightarrow$ Finance $\rightarrow 0: \sum \operatorname{Pr} n(x, y)$. Where $x$ is starting month and $y$ is number of months.

$$
\sum \operatorname{Pr} n(1,36)
$$

Results will display negative value which indicates the amount of principal paid off.

## Principal only <br> Answer - \$ 5439.70

c) To determine the total amount of Interest paid in the first 3 years

Example: Apps $\rightarrow$ Finance $\rightarrow$ A: $\sum \operatorname{Int}(\mathrm{x}, \mathrm{y})$. Where $x$ is starting month and $y$ is number of months.
$\sum \operatorname{Int}(1,36)$
Results will display a negative value which indicates the amount of interest paid over the term.

## Interest only Answer - \$ 47,391.35

d) To determine the balance of the Principal of the loan after the 3 years of payments

Example: Apps $\rightarrow$ Finance $\rightarrow 9$ : bal (x). Where $x$ is number of months you have made payments.

Bal (36)
Results will display positive value which is the principal still owed.
Answer - \$ 194,560.30
Class Discussion should include how part (d) is related to parts (b) and (c). Also include the total amount paid in three years to the Mortgage Company which would be the sum of parts (b) and (c). Further class time could be spent on discussion and discovery of changes in the Mortgage set up.
$\qquad$
Superman needs to finance his new life with Lois Lane. He thinks he will need at least $\$ 1,000,000$ for the wedding, car, house and honeymoon. He begins the search for choices in getting the money. He goes to the Metropolis Bank, Daily Planet Credit Union and to Batman, his Best Man from the Justice League of America. The chart below outlines the financial options from the three sources. Use the TI84 Financial Applications to complete the chart to make the best choice with payments due at the end of the period.

| Loan <br> Source | Rate <br> ( I ) | Payments <br> per year <br> (P//Y) | Loan <br> time | Number <br> of payments <br> (N ) * | Comp. <br> per year <br> (C/Y) | Payment <br> (PMT) | Total Loan <br> Repayment |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Metropolis <br> Bank A | $15.25 \%$ | 12 | 5 yrs |  | 12 |  |  |
| Metropolis <br> Bank B | $15.25 \%$ | 12 | 5 yrs |  | 365 |  |  |
| Metropolis <br> Bank C | $15.25 \%$ | 26 | 5 yrs |  | 12 |  |  |
| Credit <br> Union | $17.5 \%$ | 24 | 75 |  | 12 |  |  |
| Batman | $14.25 \%$ | 12 | 5 yrs |  | 12 |  |  |

*Reminder you must determine the number of payments based on the loan years and the number of payments per year.
"Finance Where?" worksheet Page 2
Discussion Questions:

1) Which loan option should Super Man choose and why?
2) Which difference in loans saves Super Man more money - changing the compounding per year or the payments per year? Justify your answer.
3) The difference between the Metropolis Bank A and Batman’s loan offerings is one percent in the interest rate. If the difference in the interest rate changes by a constant factor, does the interest change by the same constant factor rate?

Superman needs to finance his new life with Lois Lane. He thinks he will need at least $\$ 1,000,000$ for the wedding, car, house and honeymoon. He begins the search for choices in getting the money. He goes to the Metropolis Bank, Daily Planet Credit Union and to Batman, his Best Man from the Justice League of America. The chart below outlines the financial options from the three sources. Use the TI84 Financial Applications to complete the chart to make the best choice with payments due at the end of the period.

| Loan <br> Source | Rate <br> ( I ) | Payments <br> per year <br> (P/Y) | Loan <br> time | Number <br> of payments <br> ( N ) * | Comp. <br> per year <br> (C/Y) | Payment <br> $(\mathbf{P M T})$ | Total Loan <br> Repayment |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Metropolis <br> Bank A | $15.25 \%$ | 12 | 5 yrs | $\mathbf{6 0}$ | 12 | $\$ 23921.36$ | $\$ 1,435,281.60$ |
| Metropolis <br> Bank B | $15.25 \%$ | 12 | 5 yrs | $\mathbf{6 0}$ | 365 | $\$ 23970.93$ | $\$ 1,438,255.80$ |
| Metropolis <br> Bank C | $15.25 \%$ | 26 | 5 yrs | $\mathbf{1 3 0}$ | 12 | $\$ 11003.10$ | $\$ 1,430,403.00$ |
| Credit <br> Union | $17.5 \%$ | 24 | 75 | $\mathbf{1 5 0}$ | 12 | $\$ 10986.22$ | $\$ 1,647,933.00$ |
| months |  |  |  |  |  |  |  |

*Reminder you must determine the number of payments based on the loan years and the number of payments per year.

Discussion Questions:

1) Which loan option should Super Man choose and why?

Superman should choose the loan from Batman because over the length of the loan Superman would pay about \$27,000 less.
2) Which difference in loans saves Super Man more money - changing the compounding per year or the payments per year? Justify your answer.

Metropolis A \& B options are the same except for the compounding and the difference in the loan repayment was $\$ 2,974.20$

Metropolis B \& C options are the same except for the payments per year and the difference in the loan repayment was $\$ 7,852.80$
3) The difference between the Metropolis Bank A and Batman’s loan offerings is one percent in the interest rate. If the difference in the interest rate changes by a constant factor, does the interest change by the same constant factor rate?

Since the relationship is not linear, changing the difference in the interest rate by a constant factor will not affect the amount paid in interest by that same constant factor.

## Money When?

Station 1 - Spiderman

Spiderman needs to invest money to have a continuous income to purchase his supply of web materials and variety of costume every 5 years from the interest only. Determine the best investment for Spiderman using logarithmic approach for his first purchase.

## Problem 1: $\quad \$ 10,000$ invested for first 5 years.

Choice A: 4.25\% interest rate compounded monthly

Choice B: 4\% interest rate compounded continuously

Problem 2: He determines for the second 5 years he needs $\$ 3000$ in interest. At what rate does he need to find a continuously compounded investment in-order to meet his needs?

Put your answer choices and show all work on your station answer sheet as your exit from the class period.

# Money When? 

Station 2 - Robin

Robin is preparing for the engineering needs in his crime fighting by attending college for an Engineering degree. He needs a college loan to finance his studies. Determine the best option for Robin.

Tuition \$ 87000
Room and Board \$ 30000
Books and Supplies \$ 6000
Activities and Fees \$ 2000

He must begin paying 6 months after his high school graduation at the end of May in 2011. He wants to have paid off the educational loan by end of the year 2035. The rate of interest for his loan is $3 \%$ compounded daily.

Part A: Determine his monthly payment due at the beginning of the month.

Part B: How much did his education cost?

Put your answer choices and show all work on your station answer sheet as your exit from the class period.

# Money When? 

Station 3- Storm

Storm has had a stormy life and needs to plan for retirement in a nice climate. Storm is currently 22 years old. She wants $\$ 500,000$ for retirement at age 60 because her force will diminish. She has found a retirement investment that compounds annually. How much money does she need to begin to invest?

Problem 1: If she started investing now at $10 \%$ interest, what would she need to invest?

Problem 2: She needs to wait until she is 30 due to current loans she needs to pay off. With the same investment, what would she need to invest?

Problem 3: How much would she have needed to invest if she had started investing at age 18 when she had graduated from high school?

Put your answer choices and show all work on your station answer sheet as your exit from the class period.

## Money Mhea?

Station 4- Hulk
Hulk needs to purchase a new car because he mistakenly crushed his old one. He found the perfect Hummer that is big enough to handle him as either man or Hulk. The car dealership offered 3 loan options to finance the Hummer. Choose the best loan option if the Hummer purchase price including all extras, taxes, title and fees is $\$ 62599$.

Note: Each option for the Hummer loan is compounded monthly and paid at the beginning of the month.

Which option is the best choice?
Option 1: $3.15 \%$ interest for 5 years

Option 2: $\$ 2000$ down at $2.5 \%$ for 4 years

Option 3: $\$ 1000$ down at $2 \%$ for 3 years

Put your answer choices and show all work on your station answer sheet as your exit from the class period.

Group Student names
$\qquad$
$\qquad$
$\qquad$

| Spiderman Station |  |  | Robin Station |
| :--- | :--- | :--- | :--- |
| Problem 1 Circle Choice A or $\quad$ B | Part A: Monthly Pymt. $=\ldots$ |  |  |
| Show work: |  |  |  |
|  |  |  |  |
| Show work: |  |  |  |
| Show work: |  | Part B: Total Cost $=$ |  |



Superhero Comic Assignment: You are a Superhero trying to stop the crime of money mismanagement. Create a comic strip that will rescue a victim from the Credit Card repayment of $\$ 500$ at $21 \%$ interest. Credit Cards are compounded and paid monthly. Include in your comic the amount of time and the total amount the victim would have to repay the Credit Card Company if he/she was only making the minimum monthly payment of $\$ 35$.

Be creative. Use one piece of $8 \times 10$ Paper with a minimum of 4 panels. You can sketch, use computer graphics, cut and paste, etc.

Teacher Key Lesson 3 Stations Worksheet - Calculator values TMV solver "Begin"


| Spiderman Station | Robin Station |
| :---: | :---: |
| Problem 1 Circle Choice $\underline{\text { A }}$ or B | Part A: Monthly Pymt. = $\underline{\underline{\mathbf{\$ 6 0 6 . 6 6}}}$ |
| Show work: | Show work: |
| Choice A \$12,363.01 | Number of months $=289$ 12( 24 years) + 1 month |
| Choice B \$12,214.03 |  |
| Problem 2: $\quad$ Rate $=\underline{\mathbf{5 . 2 5} \%}$ | Part B: Total Cost = $\underline{\mathbf{\$ 1 7 5 , 3 2 4 . 7 4}}$ |
| Show work: | Show work: |
| Storm Station | Hulk Station |
| Problem 1 Investment amount = $\$ 13,367.43$ | Circle best Option: |
| Show work: | $1 \underline{2}$ |
|  | Work Option 1: |
| Problem 2 Investment amount $=$ $\$ 28,654.28$ |  |
| Show work: | Work Option 2: |
|  | Monthly \$ 1,325.21 |
| Problem 3 Investment amount = \$ 9,130.00 | Total \$ 65610.08 |
|  | Work Option 3: |
| Show work: | $\begin{aligned} & \text { Monthly \$ 1,761.42 } \\ & \text { Total \$ 64,411.12 } \end{aligned}$ |

