Ghost in the Mansion

A ghost was haunting a mansion on Centre Street. Although the ghost enjoyed haunting the place he occasionally became bored and therefore decided to pose the following problem to himself:

"Starting from some room can I float through all of the doors of the ground floor without reusing a door twice?"



Extensions:

"Can I find a path for the other floors in the mansion?"





The Math in This Problem:

Through the development of geometric concepts, including dimension and space, mathematicians began to study topology, a major area involving spatial properties of objects. This math puzzle involves careful, strategic planning and mathematical reasoning in order to derive a suitable solution to this maze-related conundrum.

This problem belongs to the branch of mathematics called Graph Theory. A graph bears no relation to the graphs that chart data, such as the progress of the stock market or the growing population of the planet. And you are correct, Euler was the mathematician who created this branch of mathematics. It arose from a famous problem he was working on called the Bridges of Konigsberg. Take a look at the abstract rendition of the Bridges of Konigsberg Problem to determine how to abstract the mathematical interpretation of the real situation.

To determine whether the Ghost in the Mansion problem is a Euler Path you will need to spend some time determining the properties of the path. The properties of mathematical objects (the path) form the basis for classification of those objects. It is also interesting to see the relationships between the properties that an object may or may not possess. For example a mathematician may try to find out if possessing a certain property or set of properties will always mean that there is some other property that the object is certain to have (or not have).

Here are some examples of mathematical objects and properties that they may or may not have.

- Counting numbers: Is the number even? odd? divisible by 9? prime? a square?
- Graphs: Is the graph hamiltonian? regular? complete? planar?

Hint: For the existence of Eulerian paths it is necessary that no more than two vertices have an odd degree; this means the Königsberg graph is not Eulerian.

