"I can solve problems"

5-14 Mathematics - Problem Solving & Enquiry

A supplementary resource for Secondary Mathematics

Contents

Foreword

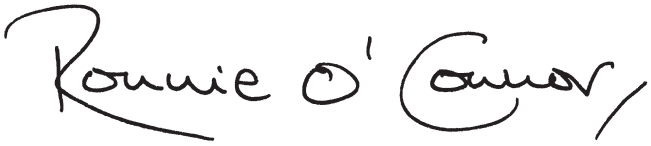
Glasgow's 5-14 Mathematics problem solving support pack *"I Can Solve Problems"* advocates a coherent, continuous and progressive approach to the development of problem solving and enquiry skills.

The pack reinforces cross curricular activities and promotes problem solving as a life skill. With increased experience of problem solving, pupils should be able to develop their thinking skills , develop characteristics such as persistence and "willingness to have a go" and acquire a toolkit of strategies. They should gain increasing confidence in approaching unfamiliar situations and this should help them in everyday situations outwith the classroom.

Interactive, participative and collaborative methods which engage pupils actively in learning are promoted and it is envisaged that these resources will enable teachers across the city to take problem solving forward in an enjoyable and dynamic way.

This pack has been developed to support and enhance the quality of teaching this aspect of Mathematics and I am sure that teachers throughout the authority will find these materials a valuable support in delivering problem solving and enquiry activities.

The materials promote a positive attitude to problem solving and all schools within the authority are encouraged to use them.



Ronnie O'Connor
Director of Education Services
Glasgow City Council

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Writing team:

Stephen Watters — Development Officer, Glasgow City Council, Education Services
Pauline Logan — St. Mungo's Academy, Glasgow City Council, Education Services

Further contributions/advice from:

Wendy O'Donnell — Adviser in Mathematics, Glasgow City Council, Education Services
Lindsay Logan — Professor Emeritus, University of Dundee
Linda Duff and pupils — St. Paul's High School, Glasgow
Members of the TFEL group, 2004-05

For permission to use materials developed within the learning community:

T. M. Burnett — St. Mungo's Learning Community, Glasgow City Council

Graphic design:

Jim French — Principal Graphic Designer, Glasgow City Council, Education Services

Art direction and design consultants:

Burns Morrison Print Management Ltd.
Buchanan Business Park, Cumbernauld Road,
Stepps, Glasgow G33 6HZ

Designer: Iain MacPherson

Produced by:

Glasgow City Council, Education Services
Wheatley House, 25 Cochrane Street,
Glasgow G1 1HL
Telephone: 0141 287 2000
www.glasgow.gov.uk

Introduction

Glasgow's 5–14 support pack I Can Solve Problems is a practical guide to assist teachers in the development of problem solving and enquiry skills.

This secondary school version is to complement the previously issued Primary I Can Solve Problems pack.

The secondary school edition problems are more age-specific for the pupils in S1 and S2, though all of the materials in the primary school edition could also be used in secondary schools.

This edition of the pack contains a condensed version of some of the advice previously issued in the primary school pack. It is a set of materials which attempts to place problem solving at the heart of mathematics and not as a bolt-on activity. It has a particular reference to 5–14 but many of the ideas and much of the advice are relevant to the teaching of mathematics beyond S2.

This pack contains:

- to supplement your existing resources, five tasks at Levels A, B and C plus ten tasks at Levels D, E and F, each with teacher notes, starters, extensions and solutions
- information on the problem solving process
- advice on effective learning and teaching
- ideas to assist with the development of pupils' awareness of strategies across levels A to E
- problem solving across the curriculum – ideas and advice.

However, the most important resource is the teacher. The creation of a learning environment which both supports and challenges pupils will enable our pupils to become confident and independent problem solvers.

Practical support for the implementation of the pack and assistance with staff development is available from Glasgow City Council Education Improvement Service.

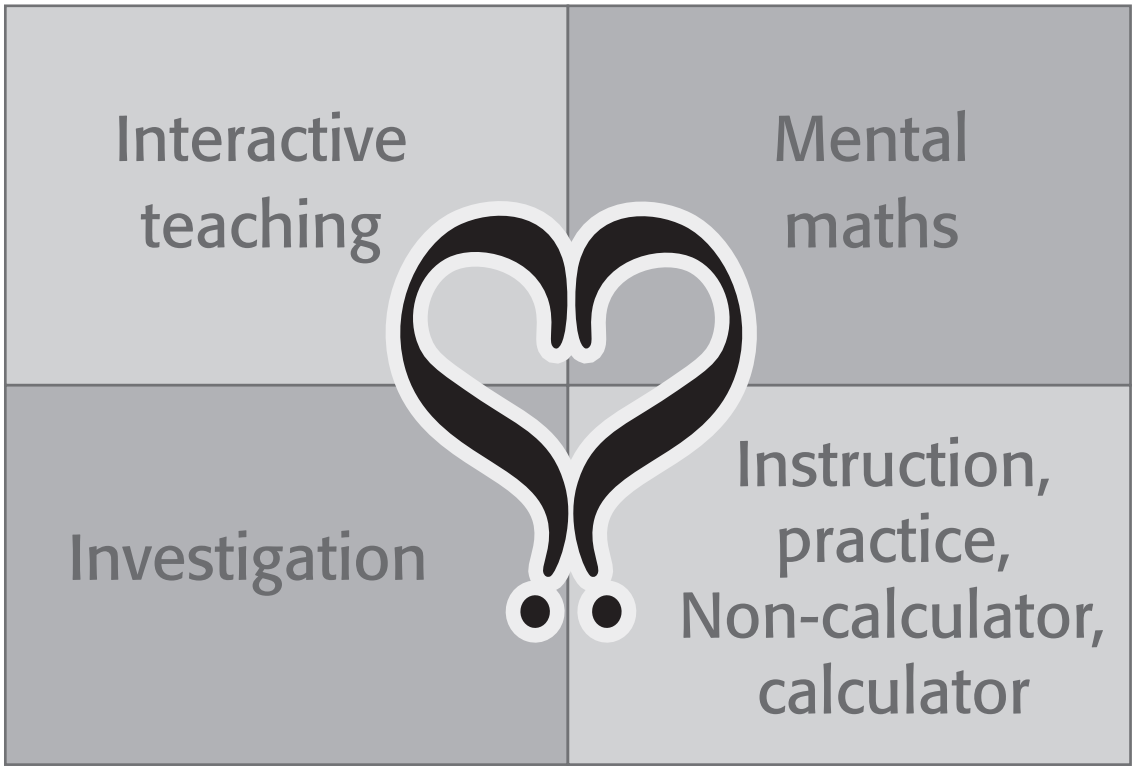
For further information concerning this pack, contact:

Wendy O'Donnell,
Adviser in Mathematics
Wheatley House
25 Cochrane Street
Glasgow G1 1HL

Section 1

Problem solving in learning and teaching

Problem solving at the heart of mathematics



The main reason for studying mathematics is to solve problems. Without the ability to solve problems, the usefulness and effect of mathematical ideas, knowledge and skills are severely limited.

The mathematics curriculum requires that a body of knowledge be retained and applied to many problems.

Problem solving is embedded in the 5–14 report, within Standard grade as Reasoning and Enquiry and throughout the National Qualification courses.

The skills taught and practised in the maths classroom also support the curriculum in other subject areas. But the problem solving techniques such as Act it Out, Draw a Diagram, Try a Simpler Case, etc are important life skills that all our pupils need in order to function in a modern society.

Teachers play an important role in the development of pupils' problem solving abilities by creating a supportive classroom environment in which pupils are encouraged to 'have a go' at solving problems. In such an environment pupils can develop confidence in tackling unfamiliar situations and problems.

Aspects of problem solving

> Solving problems is a practical art, like swimming or skiing or playing the piano: you can learn it only by imitation and practice…if you wish to learn swimming you have to go into the water, and if you wish to become a problem solver you have to solve problems.
>
> From Mathematical Discovery by George Polya

In order to become skilled, confident problem solvers pupils have to be given the opportunity to apply their thoughts and ideas to solve problems and through doing so (and with the support and guidance from the teacher) build up an awareness of problem solving strategies which will give them the confidence to meet future challenges.

Opportunities to develop pupils' problem solving abilities occur in most maths lessons and not just in stand - alone problems. Pupils will become more confident problem solvers if there are an increased number of occasions when they are asked to use and explain problem solving strategies.

The 5–14 guidelines describe the three main aspects of problem solving as:

a. adopting an investigative approach to learning concepts, facts and techniques

b. working on tasks designed specifically to highlight the merits of certain approaches to mathematical thinking

c. using mathematics in an enquiry that could be part of a cross-curricular study.

This pack contains tasks specifically designed to highlight certain problem solving techniques, but the teaching, learning and understanding will be enhanced if these tasks are complemented by further integrating problem solving methods in the curriculum and in day-to-day lessons.

Adopting an investigative approach to learning concepts, facts and techniques

One way to encourage pupils to think mathematically for themselves is to adopt an investigative approach to learning, where pupils are given time, space and resources to find things out for themselves while the teacher plays a supportive encouraging role.

While concerns regarding time constraints and coverage of the curriculum should be recognised, it is generally accepted that children's understanding and retention will be strengthened as a result of an investigative approach, and so opportunities should be incorporated into the child's learning experience.

With different age groups and levels of attainment, the amount of teacher intervention will vary, but at all stages children benefit from being asked to find out for themselves.

This approach can also be used to consolidate previous knowledge.

A few possible ideas:

1. Exploring the sum of the angles in a triangle by cutting them out. Can this method be used for other shapes?
2. Comparing the differences and similarities of 2-D and 3-D shapes. For example, what is the same and what is different about a rhombus and a parallelogram?
3. What patterns can be found in the times tables?
4. Can the rules for areas and volumes be discovered through practical activities?
5. Statistics taught through pupil surveys.
6. Learning the properties of 3-D shape through model building.
7. Rules for multiplying and dividing by 10, 20,100, 1000, etc by guess and check or by spotting patterns.
8. Can the pupils devise different methods for finding percentages? For example, find three ways to calculate 25% of 420. Which is the best method? Is it always the best method? Why?
9. The equation of straight lines by spotting patterns from a table.
10. Can pupils find rules for adding and subtracting negative numbers by spotting patterns?
11. Find two different methods to multiply 23 by 67. Explain which method is easiest. Would the same method be as effective for 212 multiplied by 31?
12. Mathematical charades. Pupils take it in turn to give clues to any mathematical shape or concept and the class use trial and error or elimination to find the correct shape. For example: I am a four-sided shape, I do not have right angles in the corner, etc.

Many classroom activities can be adapted to allow pupils further opportunities to engage in problem solving activities. If undertaken in pairs or groups this can promote mathematical discussion and enhance understanding.

Working on tasks designed specifically to highlight the merits of certain mathematical thinking

This pack contains 5 stand-alone problems at Levels A, B and C as well as 10 stand-alone problems at each of the levels D, E and F. These are stand-alone, non-routine questions for which the answer can be arrived at by using a variety of strategies.

There are many commercially produced packages being used in schools at present and these materials should complement the resources already used in schools.

At each level the problems are designed so that pupils can experience a range of problem solving strategies.

Although the problems are designed to cover different strategies, pupils should be encouraged to solve the problem using whatever strategy they see as suitable.

The plenary session is ideal to reinforce the strategies used, but also allows pupils to discuss what was the most effective method for solving the problem and what strategies they would use if they were to tackle a similar problem.

Using mathematics in an enquiry or in cross-curricular studies

This type of open-ended enquiry can often usefully address problems identified by the students themselves, or may be a theme-based investigation.

Although there are more opportunities in the Primary classroom for this type of approach it can be incorporated in a Secondary maths curriculum, perhaps as part of an enterprise project or fundraising events.

In this type of enquiry the boundaries between mathematics and other curricular areas will often be blurred, as indeed they usually are in real-life situations.

Learning and teaching in the problem solving process

The teacher's role

Teachers should:

- encourage pupils to have a 'can do' or 'can try' attitude. Pupils should expect to struggle, to be 'stuck', and not always to obtain the correct answer. They also need to realise that success can be the attempt at the problem and the learning experience gained. The discussion – and use –of success criteria prior to starting on the problem would assist many pupils to realise that finding a correct answer is not the only way to be successful.
- create a supportive classroom where pupils are prepared to tackle the unfamiliar and not feel threatened when they get stuck – a classroom that allows and encourages peer support.
- allow pupils to follow their own ideas. Teachers should assist when appropriate without giving the answer away. Pupils need to be allowed to struggle with a problem to gain confidence in their own abilities.
- provide space in the lessons for pupils to reflect on their own thinking by discussing with a partner or by reporting back to a group or class and so deepen their understanding of the processes involved.
- talk to the pupils about the processes so that they can build up a vocabulary and so that pupils are explicitly aware of the strategies involved.

Hints for effective classroom practice

Provide opportunities for pupil interaction

Much of the latest research indicated that we learn more from what we discuss with others or explain to others than from just listening or reading. So we should aim for structured lessons that are balanced in terms of reading, listening and interaction (with materials, teacher and other pupils).

In the problem solving materials provided there are many opportunities for the pupils to interact with fellow pupils, resources and the teacher.

It is suggested that the problems are best tackled in pairs or small groups to allow for pupil–pupil interaction and to allow the pupils to support each other through the challenges. For example: 'Talk with your partner for two minutes. Discuss the best strategies you could use to tackle this problem.'

Some of the problems also give opportunities for the class to act out the problem or make models. Both of these strategies would enhance the learning experience of the students and lead to deeper understanding of the problems.

Share the purpose of the lesson

Clearly stating the purpose of the lesson allows the pupils to connect to previous learning and to use that learning as the basis of the new learning. For example: 'During today's lesson we will be learning to…(e.g. solve problems). What did we say was the first step to solving problems?'

Pupils should be aware of what you expect of them

For example: 'Today you will have been successful if

a. you have discussed, with your partner, how to tackle the problem

b. you have clearly laid out your working

c. you can report back to the class the strategies you used and why these were the best strategies to use.'

The success criteria could also be broken down into small chunks in the lesson if the class requires further support. For example: 'For the next ten minutes you should…'

If you clearly state what you expect of the pupils and how they can be successful, they are more likely to stay on track and be further motivated to attempt unfamiliar challenges.

Make effective use of questioning

Pupils need to be challenged to think and talk about what they are doing when solving problems. The effective use of a blend of lower order and higher order questioning will help stimulate deeper understanding of the processes involved in solving the problem.

Some questions to develop thinking:

Getting started

- Has anyone seen a problem like this before?
- What are the important ideas/pieces of information in this problem?
- Can you rephrase (express…) the problem in your own words?
- What is the problem asking you to find out?
- What information has been given?
- Can you guess what the answer might be?
- What strategies might you use to get started?
- Which of these ideas are worth doing?

While working on the problem

- Tell me what you are doing.
- Why (How) did you think of that?
- Why are you doing this?
- What will you do with the result of that work when you've got it?
- Why is this idea better than that one?
- You've been trying that idea for five minutes. Is it time to try something else?

At the finish

- Have you answered the question?
- Have you considered all possible cases?
- Have you checked your solution?
- Does the answer look reasonable?
- Is there another answer?
- Is there another solution?
- Can you explain your solution to the rest of the class?
- Can you extend the problem further?

Please note that the wording of these questions should be amended according to the level /maturity of the pupil.

Pupils' responses need to be appreciated as having value even when not correct. They can then be opened up to other pupils to see if they agree and so stimulate further discussion. Valuing all responses will encourage pupils.

Lesson structure

A suggested lesson structure would be:

a. lesson starter (5 mins)
b. introduce problem
c. pupils discuss possible strategies and start (5–10 mins)
d. teacher stops class and checks on progress, and intervenes if necessary (5 mins)
e. pupils attempt problem, teacher supports individual pupils (15 mins)
f. plenary – discussing the main strategies used in the problems (5–10 mins).

Lesson starter

An activity that allows the pupils to 'warm up' their thinking.

The starter is meant to be quick-fire to engage and motivate the pupils from the beginning. Some of the starters are designed to remind pupils of possible strategies that could be used to tackle the problems.

Learning intention and success criteria

As previously stated, sharing the purpose of the lesson and clearly stating how pupils can be successful can support the teaching and learning process.

The focus of the work is on learning how to tackle problems, and not on the need to obtain the correct answer. This should be stated clearly in the learning intentions and success criteria. Success for the pupils is the effort and the process of thinking, discussing, attempting, and reporting on the problem. This should be stated explicitly at the outset.

Introduction

The teacher gives a brief introduction to the problem to ensure that the concept of the task and the vocabulary in the task are understood. This could include asking the pupils to describe the task in their own words. The teacher should not offer pupils methods for solving the problem at this stage. The pupils are being asked to do the thinking.

Thinking time

Pupils can then be allowed to discuss the task in pairs or groups and come up with possible strategies to attempt the problem. This thinking time, allowing pupils time to struggle with the problem, is essential if we wish pupils to develop confidence in their own problem solving abilities and to prepare them for Reasoning and Enquiry questions in S3, 4, and 5. However, teachers should intervene and give direct help, if necessary, to avoid pupils being overwhelmed or frustrated.

It would be supportive to the pupils if teachers had discussed, prior to the problem solving work, methods for pupils to overcome difficulties themselves. (See Section 5, What do I do if I'm stuck?)

Teacher intervention

After allowing the pupils approximately 5–10 minutes to discuss and start the problem it is suggested that the class is brought together and the progress made discussed. This is an opportunity to support the pupils further by discussing any difficulties being encountered and clarifying some strategies that may be useful in tackling the problem.

Doing

Pupils are given further time to attempt to solve the problem. The teacher can intervene whenever required. Guidance on recording work and on how to lay out the work could be helpful.

Plenary

The lesson(s) should be completed by allowing the pupils opportunities to report back on the main strategies used, discuss any areas of difficulty, review the solutions obtained and evaluate whether solutions are relevant.

This is an opportunity to review the different methods that may have been used and to allow the pupils to decide if there is a particular strategy, or strategies, that are the most efficient in producing solutions.

Plenary sessions will also assist pupils retain knowledge over a longer period and support the next problem solving activity.

It is important in the learning process that the plenary session is included.

The teacher should stress that how a problem is solved is important. Pupils are learning techniques that can be used in other parts of maths and in other subjects. This emphasis on the process enables pupils to experience some success in problem solving no matter the answers obtained.

It is the effort and the processes of thinking, discussing, attempting, and reporting back that are the main focus of the activity and this should be recognised by the teacher.

Help pupils with strategies for recording and presenting their work.

Many pupils will require support to record and present their work in a neat and organised fashion. Some of the strategies used to solve the problems assist this, for example make an organised list, make a table or draw a diagram.

There are also templates for record sheets provided in section 5 of the Primary Problem Solving Pack.

The problem solving process

The problem solving process can be broken down into three steps:

- understanding and planning
- solving
- communicating and evaluating.

Understanding and Planning

The first step in solving any problem is to understand it. Pupils should be introduced to the problem, given opportunity to discuss it, decide which information is important and if any vocabulary is unfamiliar. Pupils should then plan how to proceed by considering if they have tackled similar problems and what strategies could be used

Hints for pupils

- Read the problems two or three times
- Check with your partner that you both agree what questions is asking you what to do.
- Visualise the problem.
- What do I know? What do I want to know?
- Have I tackled a similar problem?

Solving

Hints for pupils

- What strategies could I use?
- Write down what you do.
- Check your answer – does it make sense?
- If stuck, try a different method.

Communicating and evaluating

When pupils are given the opportunity to explain their solution, to a group or the class, their understanding of the problem will deepen.

The class discussion should try to focus on the different methods used and the merits of one strategy compared to another.

It would assist retention if the methods used were listed, by the teacher, as the pupils explain them.

The evaluation could include:

- Does the answer make sense?
- Could more efficient methods be used to tackle the problem?

The teacher should stress that how a problem is solved is important – not just finding a correct answer. An emphasis on the process enables all pupils to experience some success in problem solving. The effort, rather than the answer, should be recognised as much as possible.

Developing a guide for progression

Here is a list of some of the main strategies that can be used to solve problems.

You will find these listed on page 13 of the 5-14 National guidelines although the strategy 'work together' has been added to ensure that we give priority to collaborative work in solving problems.

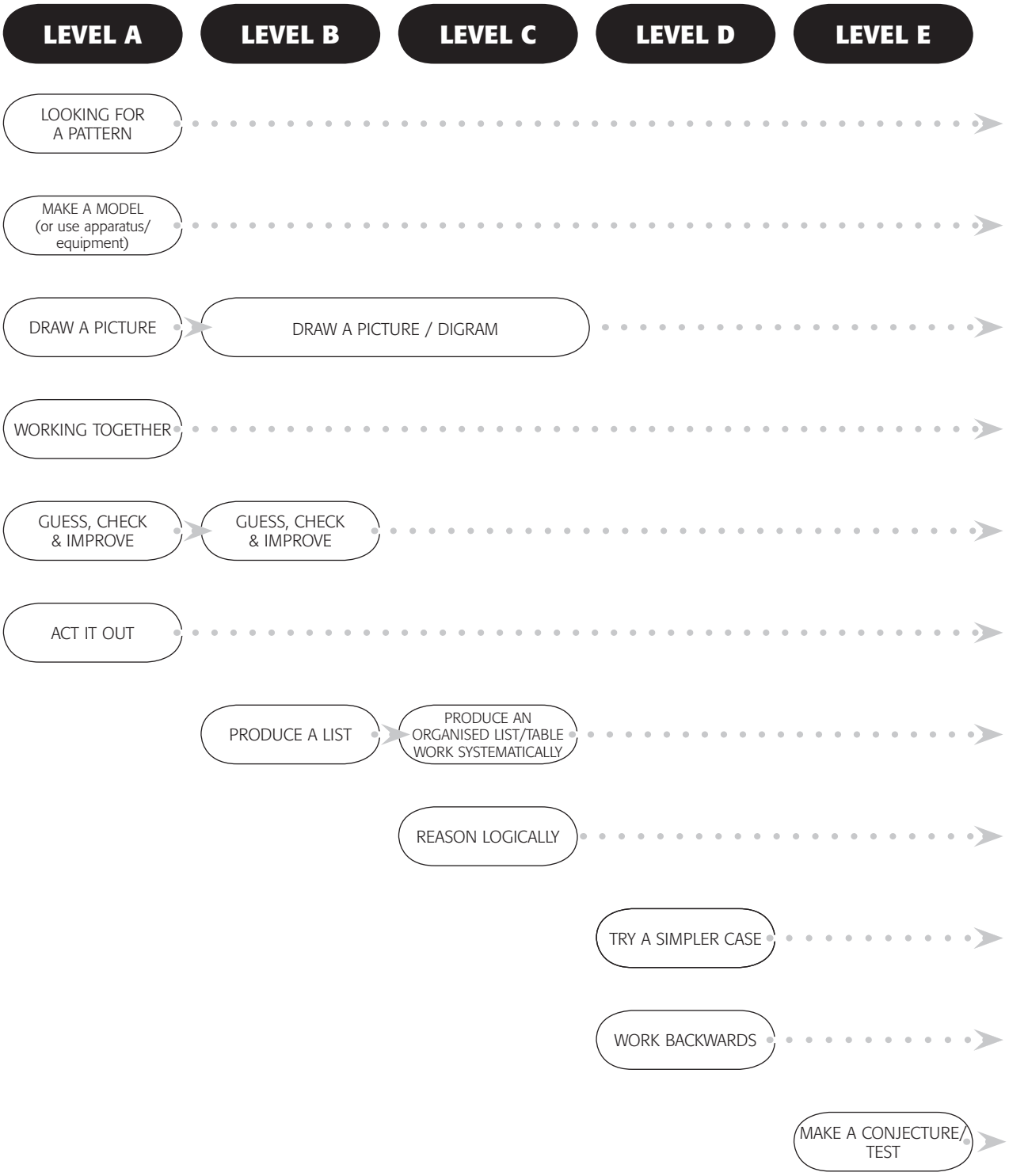
- Look for a pattern
- Make a model (or use apparatus/equipment)
- Draw a picture/diagram
- Work together
- Guess, check & improve
- Act it out
- Produce an organised list/table
- Reason logically
- Try a simpler case
- Work backwards
- Make a conjecture & test it with particular examples

To ensure that pupils gradually become skilled in applying these strategies, a guide has been produced to systematically introduce these strategies as the pupil progresses. Further explanations can be found in the notes accompanying 'Developing strategy - a guide for progression' included in this section of the pack.

On the following page you will find a guide which gives a structure introducing the key strategies at different stages.

This guide has been written to allow pupils to be introduced to different key strategies at different stages enabling pupils to gradually become more competent and confident in problem solving. It seems that pupils are able to tackle and use more strategies as they continue with problem solving and they are also able to use them to a deeper level. It is important to emphasise that this programme is not definitive and possibly younger pupils can effectively use other strategies, but it is important to introduce them gradually in order to build pupils' confidence and perseverance. As problem solving becomes much more integrated into the curriculum, some of the more 'difficult' strategies might come into use earlier on. Therefore this guide can be evaluated and amended in light of experience. Should you feel that your pupils are ready to meet any of the strategies earlier than expected or that their use crops up naturally in teaching, then please feel free to introduce them. The key strategies help give us a structure but it is important to note that some problems require pupils to use a variety of strategies. It is important to note that an over emphasis on pupils using the strategy that is advised could prove detrimental to pupil experimentation and creativity and pupils should be allowed the freedom to explore and discuss various ways of tackling the problems - ideas that worked and ideas that didn't.

We must also take into consideration the personal skills and attitudes to problem solving that we wish to develop and so a separate guide, showing the development of skills/attitude, has also been devised to be used in parallel with this. Please see guide further on in this section.


LEVEL A
LEVEL B
LEVEL C
LEVEL D
LEVEL E
LOOKING FOR A PATTERN
MAKE A MODEL (or use apparatus/ equipment)
DRAW A PICTURE
DRAW A PICTURE / DIGRAM
WORKING TOGETHER
GUESS, CHECK & IMPROVE
GUESS, CHECK & IMPROVE
ACT IT OUT
PRODUCE A LIST
PRODUCE AN ORGANISED LIST/TABLE WORK SYSTEMATICALLY
REASON LOGICALLY
TRY A SIMPLER CASE
WORK BACKWARDS
MAKE A CONJECTURE/ TEST

Problem solving programme

It is suggested that the problem solving materials are incorporated into the 5–14 curriculum in a planned way, so that all pupils have the opportunity to experience challenging and appropriate problems as they progress through the school year.

The Glasgow 5–14 Programmes of Study have, as well as embedded problem solving activities, designated time slots for problem solving experiences. The tasks provided in this pack, along with materials already used in schools, could provide the resources required.

The tasks in this pack, which are designed to highlight the use of particular strategies, could be attempted at the end of each topic or every two to three weeks.

Section 2

Problem solving tasks and lesson plans

Level A

Contents

Teachers Notes

Resources

Each lesson has:

1. lesson starter materials
2. pupil worksheets
3. teacher notes
4. solutions.

Teacher notes highlight the main strategies that could be used in the problems.

The lesson starters are designed, whenever possible, to be related to the problem that will follow by highlighting possible strategies that could be used in the lesson and also to review vocabulary used in some problems.

Overheads are included for some problems to clarify diagrams or to allow games to be played by the teacher with the class.

The order in which the problems are tackled is not important.

Class structure

It is suggested that all of these problems should be tackled with the pupils working in pairs or small groups. Learning to reason requires interaction with someone else.

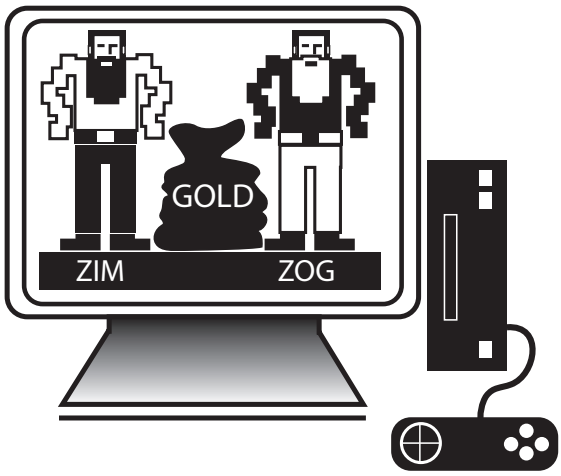
Pupils need to be challenged to think and talk about what they are doing when solving problems.

It should be stated explicitly to the class why they are in pairs or groups. It is so that they can discuss the problem and so that they can support each other. It is not just by accident that they have a partner.

Problems are best solved by pupils discussing with each other the possible strategies that can be used to tackle the problem and by supporting each other through the solving process.

Most pupils will learn more and be more confident attempting the problems using a paired or group approach rather than having to solve the problems on their own.

Zim and Zog



Jane and Mo are playing a computer game.

The game needs them to share out gold coins between 2 giants, Zog and Zim.

When there are 2 coins Jane says, 'We can share out the coins 3 ways.'

ZOG

ZIM

0 COINS

ZOG

ZIM

ZOG

0 COINS

ZIM

The problems

a. The next level of the game is to share out 4 gold coins. How many different ways can this be done?

b. The final level to the game is to share out 10 coins. How many different ways can this be done?

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Draw a diagram
b. Guess and check.

RESOURCES Counters.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Two numbers add to 6.
What numbers are they?
How many different answers are there?

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
If possible pupils could act out the problem with coins. This would assist a deeper understanding.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

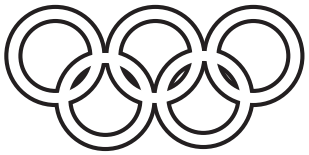
PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION Bring the class together and allow several pupils to report their solutions. Discuss any difficulties.

a. 0 and 4; 1 and 3; 2 and 2; 3 and 1; 4 and 0

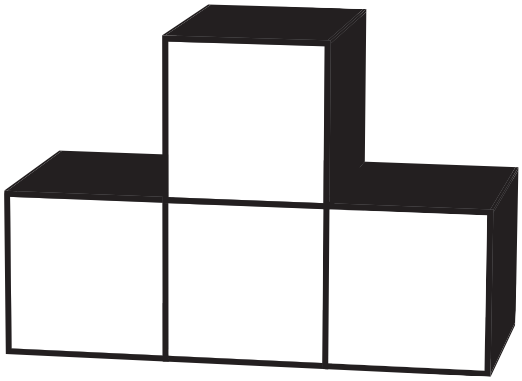
b. 0 and 10; 1 and 9; 2 and 8; 3 and 7; etc

Olympic Medals



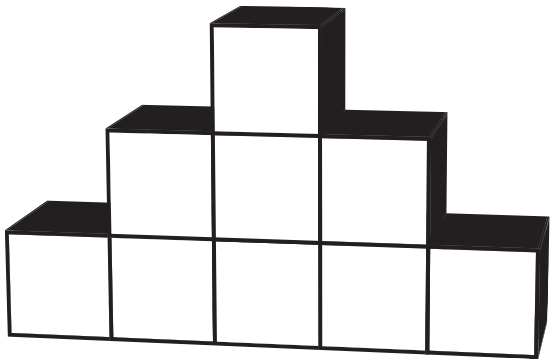
At the Olympic Games the winners stand on boxes to get their medals.

For three medal winners four boxes are needed to give the medals.
Use your cubes to make this shape.



The problems

a. So that 5 people can get medals more boxes are added.



Use your cubes to make this shape.
How many cubes did you need to make the shape?

b. For 7 people to get medals more boxes again will be added.
Use your cubes to make the boxes for 7 people.
How many cubes did you need to make the shape?

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Make a model
b. Look for a pattern

RESOURCES Cubes are needed for each child/pair.

TIME 1 period

LESSON STRUCTURE a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Find the odd one out from: 9, 10, and 11.
There is more than one possible answer.
Pupils could be encouraged to explain their answer.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem**.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

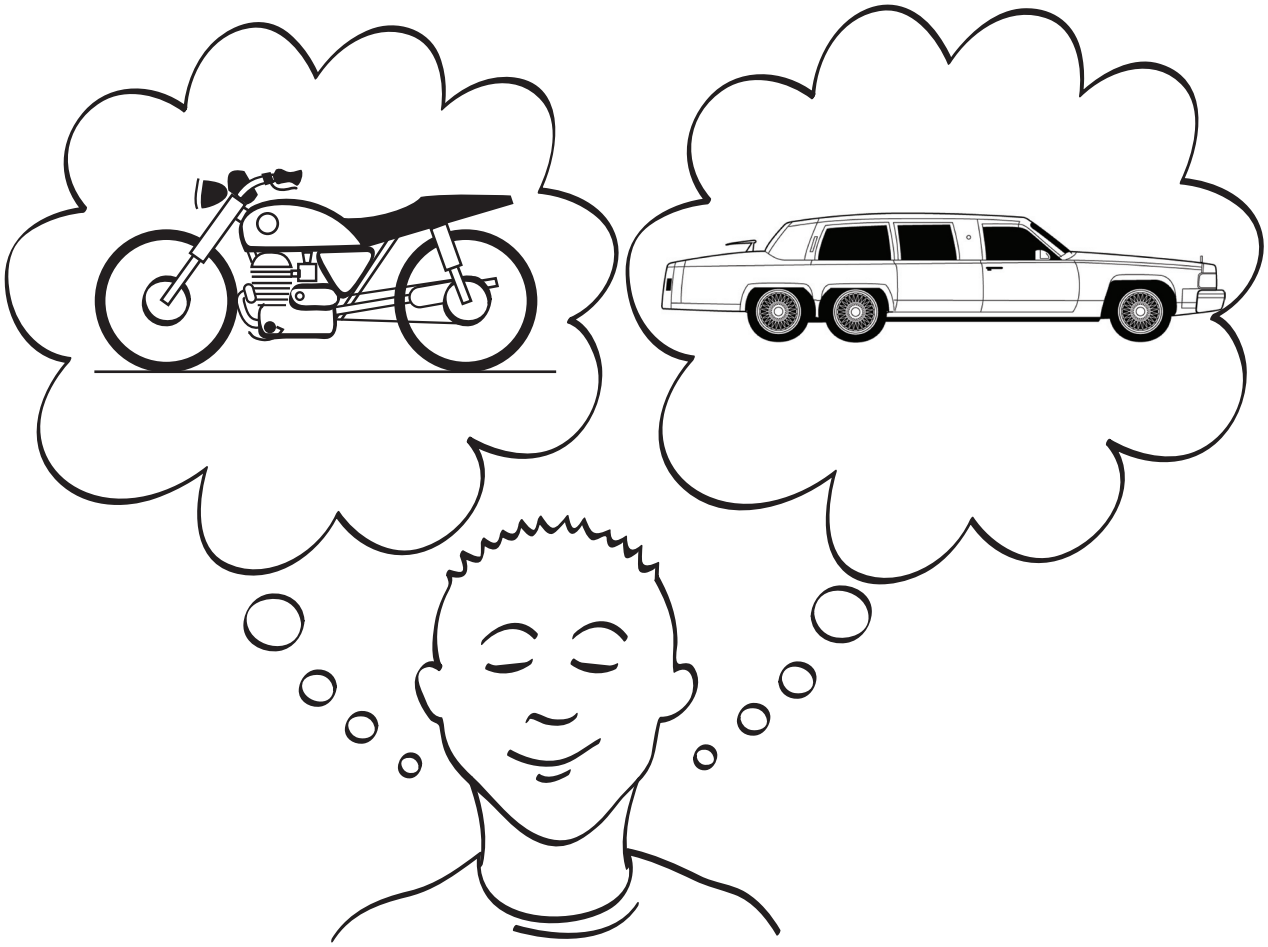
PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION Bring the class together and allow several pupils to report their solutions. Discuss any difficulties.

a. 9 cubes

b. 16 cubes

Motorbike Madness



Ali is mad for motorbikes and limousines.

He knows that motorbikes have 2 wheels.

He also knows that limousines have 6 wheels.

During a boring lesson he looks out of window and is amazed to see a number of motorbikes and limousines.

In total he counts 18 wheels.

How many motorbikes and limousines did Ali see?

Could you find more than one answer?

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Make/use a model
b. Guess and check

RESOURCES If possible, model cars/bikes or plasticine and counters are needed to make models.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP The answer is: 8
How many different questions can you come up with that will give you this answer?

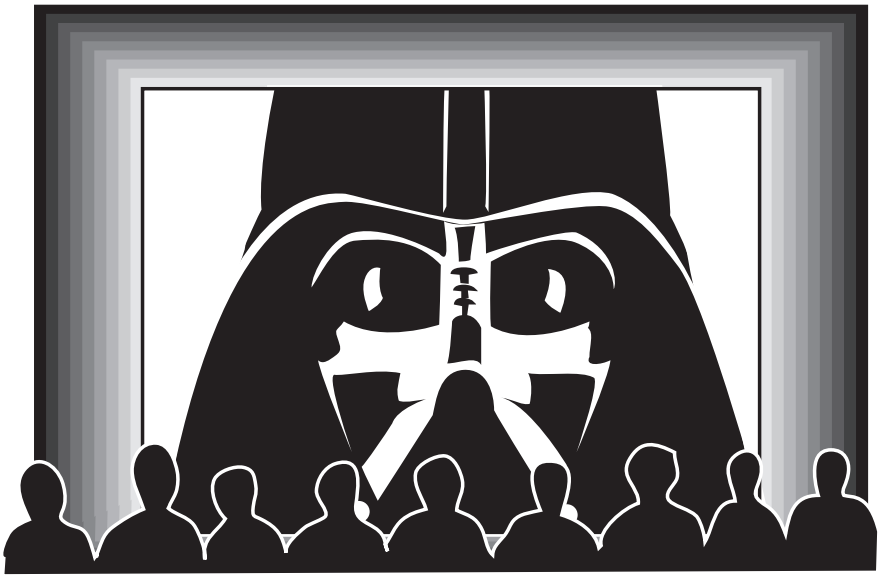
STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
If possible pupils could make or use models.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION Bring the class together and allow several pupils to report their solutions. Discuss any difficulties.
3 L + 0 B; or 2 L + 3 B; or 1 L + 6 B; or 0 L + 9 B: with the middle two answers having both vehicles.

Cinema Visit



A puzzle for four pupils

Ayesha, Brian, Carol, and Darius go to the cinema.

Use the clues to decide where they sit.

a. Ayesha does not like Brian.
b. The boys want to sit together.
c. Carol doesn't like Brian either.
d. Darius is sitting between a boy and a girl.
e. Ayesha is sitting on the seat at the far left of the group.

Fill in the seats where all the pupils sit.

Seats

Make up your own puzzle. Write it down.

Ask another group or another pupil to solve it.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Act it out
b. Guess and check

RESOURCES Four cards are required, with the names A, B, C, D on them, to allow pupils to guess and check solutions.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Two numbers add to 6.
What numbers are they?
How many different answers are there?

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
If possible pupils could act out the problem in groups of four. This would assist a deeper understanding.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION Bring the class together and allow several pupils to report their solutions. Discuss any difficulties.
Ayesha, Carol, Darius, Brian

Cheerleaders

The local cheerleading team are using red, blue and green for their uniform.

The uniform is a hat, a t-shirt and shorts.

The uniform must use red, blue and green for only one piece of clothing.

Here are two different ways the uniform could be made.

HAT	Green
T-SHIRT	Red
SHORTS	Blue

OR

HAT	Red
T-SHIRT	Blue
SHORTS	Green

Pupil Worksheet

Colour the worksheet to show all the different ways the cheerleaders could choose their uniform.

HAT	
T-SHIRT	
SHORTS	

HAT	
T-SHIRT	
SHORTS	

HAT	
T-SHIRT	
SHORTS	

HAT	
T-SHIRT	
SHORTS	

HAT	
T-SHIRT	
SHORTS	

HAT	
T-SHIRT	
SHORTS	

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Act it out
b. Look for a pattern

RESOURCES If possible three different coloured sets of clothing are required.
Pupils need coloured pencils.
Pupil worksheet.

TIME 1 period

LESSON STRUCTURE a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP The answer is 12.
Make up as many sums as possible with this answer.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and allow the pupils to formulate their own strategies for solving the problem.
If possible pupils could act out the problem with different coloured clothing. This would assist a deeper understanding
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

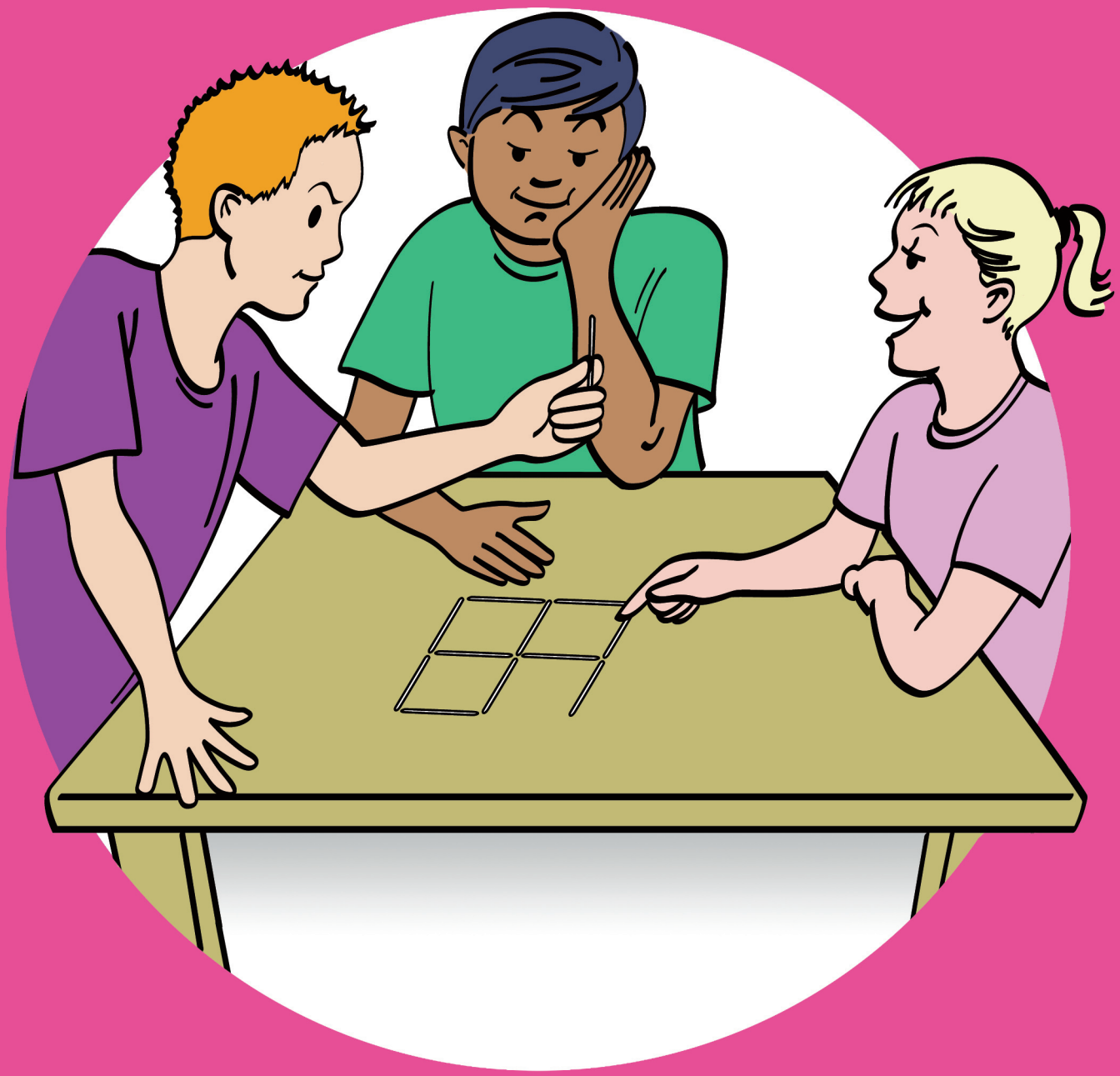
PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION Bring the class together and allow several pupils to report their solutions. Discuss any difficulties.
RGB, RBG, BGR, BRG, GRB, GBR

Section 3

Problem solving tasks and lesson plans

Level B



Contents

Teachers Notes

Resources

Each lesson has:

1. lesson starter materials
2. pupil worksheets
3. teacher notes
4. solutions

Teacher notes highlight the main strategies that could be used in the problems.

The lesson starters are designed, whenever possible, to be related to the problem that will follow by highlighting possible strategies that could be used in the lesson and also to review vocabulary used in some problems.

Overheads are included for some problems to clarify diagrams or to allow games to be played by the teacher with the class.

The order in which the problems are tackled is not important.

Class structure

It is suggested that all of these problems should be tackled with the **pupils working in pairs or small groups**. Learning to reason requires interaction with someone else.

Pupils need to be challenged to think and talk about what they are doing when solving problems.

It should be stated explicitly to the class why they are in pairs or groups. It is so that they can discuss the problem and so that they can support each other. It is not just by accident that they have a partner.

Problems are best solved by pupils discussing with each other the possible strategies that can be used to tackle the problem and by supporting each other through the solving process.

Most pupils will learn more and be more confident attempting the problems using a paired or group approach rather than asking them to solve the problems on their own.

Blind Date

A hundred S2 pupils are playing a game .The pupil who wins the game gets to choose who they will sit beside in class.

Each pupil is given a card with a number and all the pupils stand up.

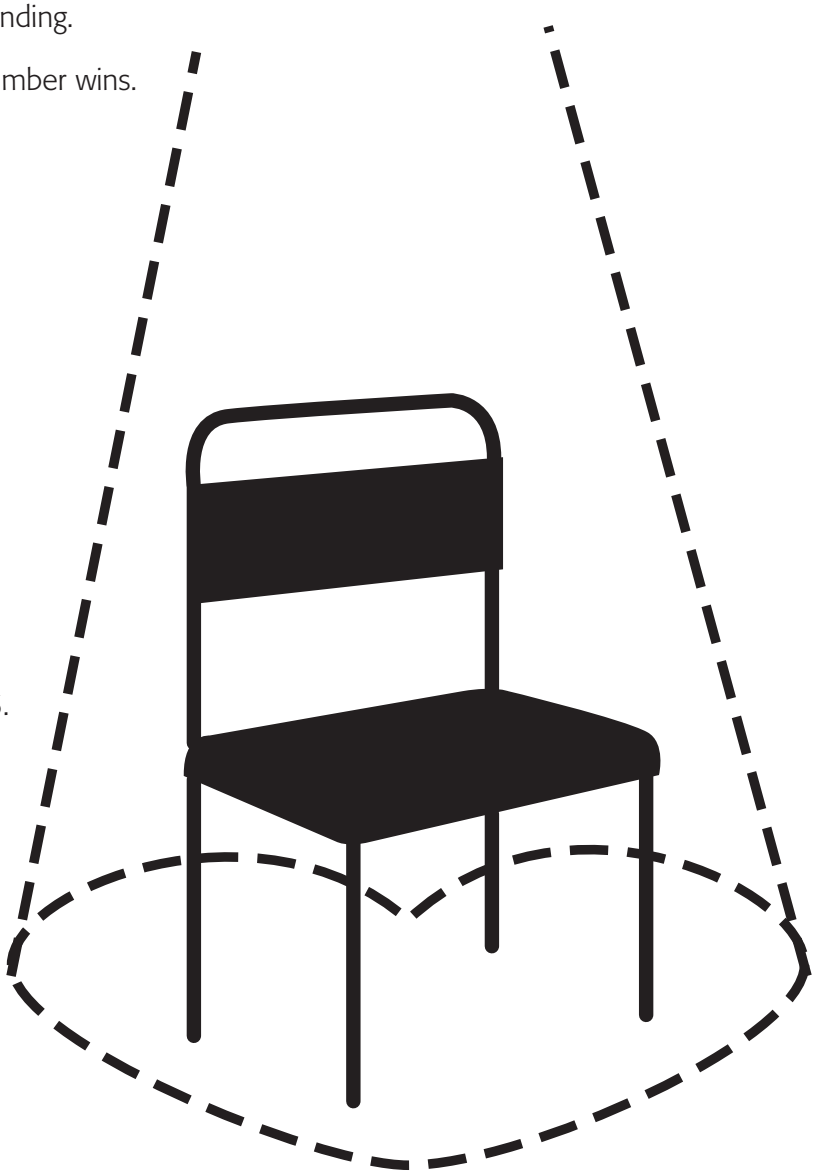
The numbers on the cards are from 1 to 100.

The teacher gives clues.

If the number each pupil has does not agree with the clue then they have to sit down.

The winner is the pupil who is left standing.

From the clues decide which pupil number wins.



Game 1

The number has two digits.
The number is odd.
The sum of the two digits is 5.
The digits differ by one.
Which number wins?

Game 2

The number is greater than 50.
The number is an even number.
If you add the digits you get 13.
The difference between the digits is 3.
Which number wins?

Game 3

Can you make up your own game?
Give the clues to your partner and
see if they can they solve the puzzle.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Elimination

RESOURCES A number grid from 1 to 100 is needed.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Find the sum of the digits for the number 567.
What is the difference between the digits for the number 93?

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and allow the pupils to formulate their own strategies for solving the problem.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

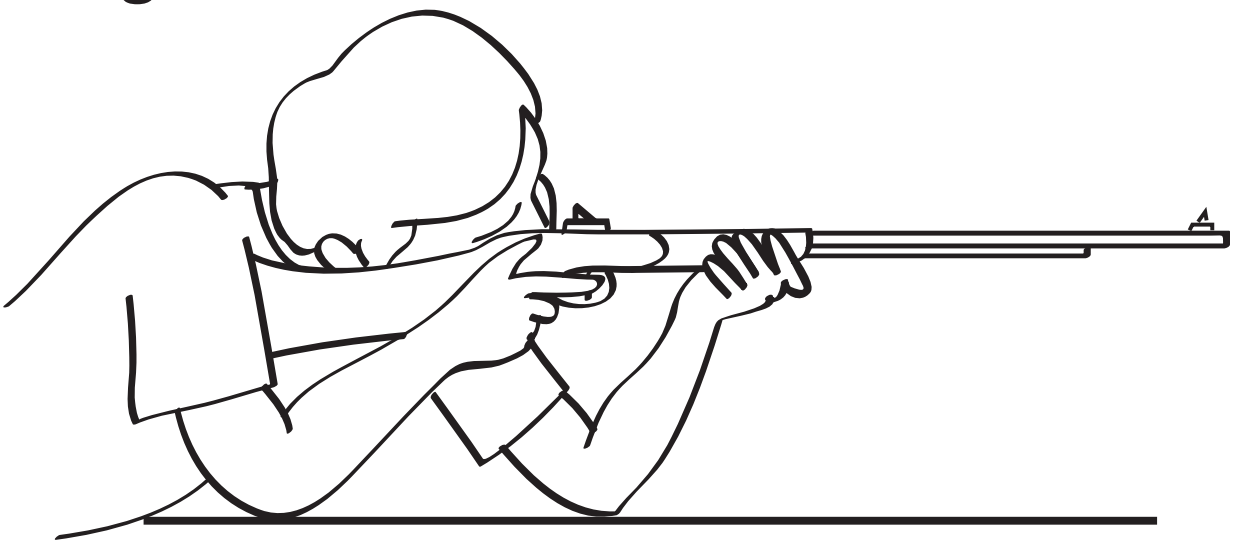
PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION Bring the class together and allow several pupils to report their solutions. Discuss any difficulties. The use of elimination as a problem solving strategy should be emphasized.
Game 1: 23
Game 2: 58

Blind Date Number Grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

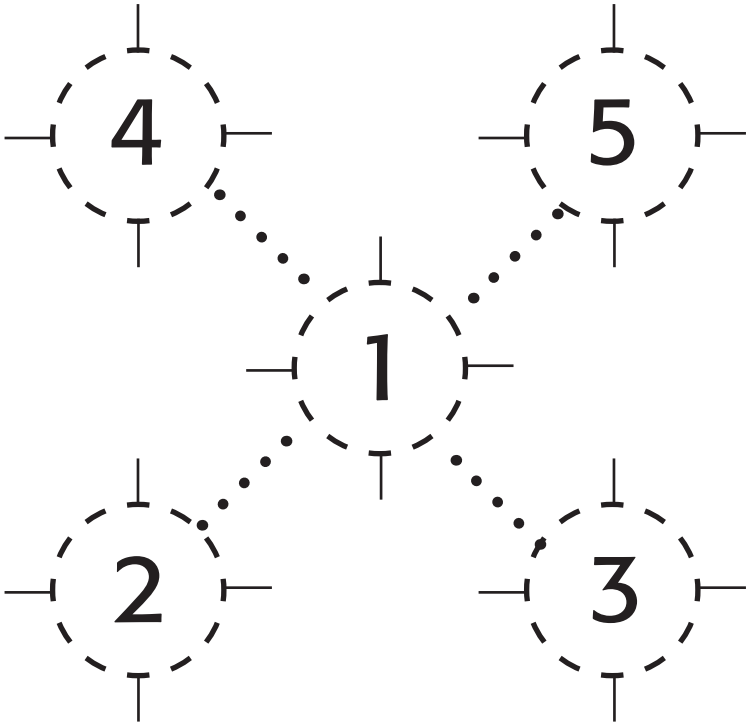
Fairground



Raza and Jo go to the fairground to try to win some prizes at the shooting gallery.

To win they must hit targets a certain number of times.

Both the crossed lines of the target must have the same total.

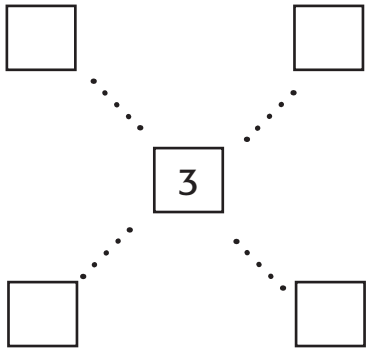


This wins because both lines add to 8.

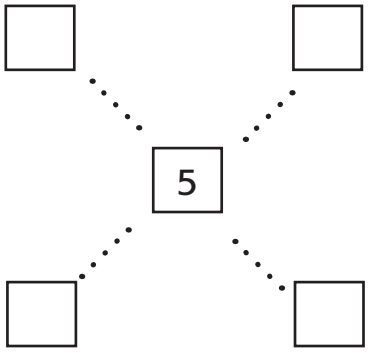
Complete the targets on the worksheet so that Raza and Jo win.

Pupil Worksheet

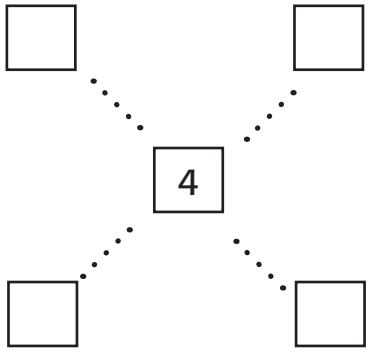
1. Digits 1–5, total 9



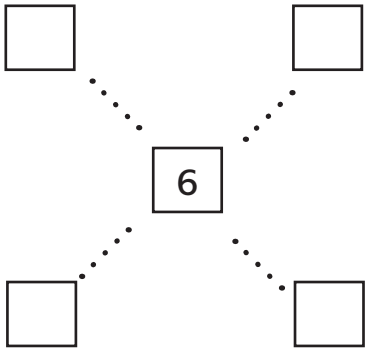
2. Digits 1–5, total 10



3. Digits 2–6, total 12



4. Digits 2–6, total 13



Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Guess and check and improve

RESOURCES Pupil worksheet.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Find the sum of the digits for the number 34.
How many digits does this number have: 34501?

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

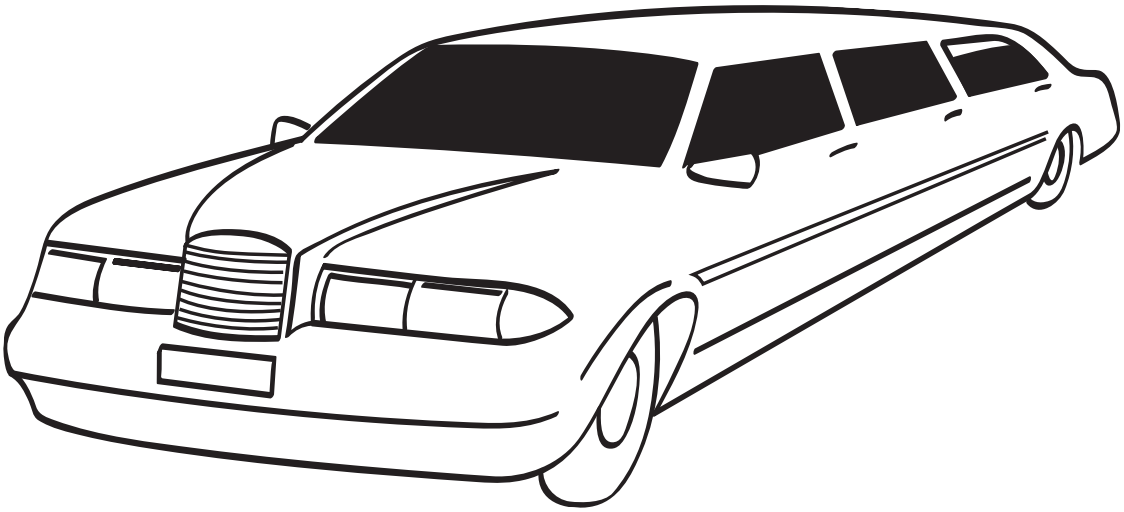
DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION Bring the class together and allow several pupils to report their solutions. Discuss any difficulties. The use of guess and check as a problem solving strategy should be emphasised.
Possible solutions:

1. 234 and 135
2. 253 and 154
3. 246 and 345
4. 265 and 364

Limousine



Davi, Edi, Farah and Ger hire a limousine to take them to the school prom.

The back seat is big enough to seat all four pupils. However, they cannot decide who will sit where. So every minute they change seats.

Here are two ways they could sit.

Seats

DAVI	**FARA**	**EDI**	**GER**
FARA	**EDI**	**GER**	**DAVI**

Find as many more ways as you can that the pupils can sit together.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Act it out
b. Guess and check
c. Look for a pattern

RESOURCES Four cards are required, with the names D, E, F, G on them, to allow pupils to guess and check solutions.
Children could work in fours to act out the problem.

TIME 1 period

LESSON STRUCTURE a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Two numbers add to 10 but multiply to 24.
What numbers are they?

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and allow the pupils to formulate their own strategies for solving the problem.
If possible pupils could act out the problem in groups of four. This would assist a deeper understanding.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION Bring the class together and allow several pupils to report their solutions. Discuss any difficulties. Did any pupils spot any patterns?
There are 24 possible solutions The pupils are not expected/asked to find them all.

Odds and Evens

A puzzle

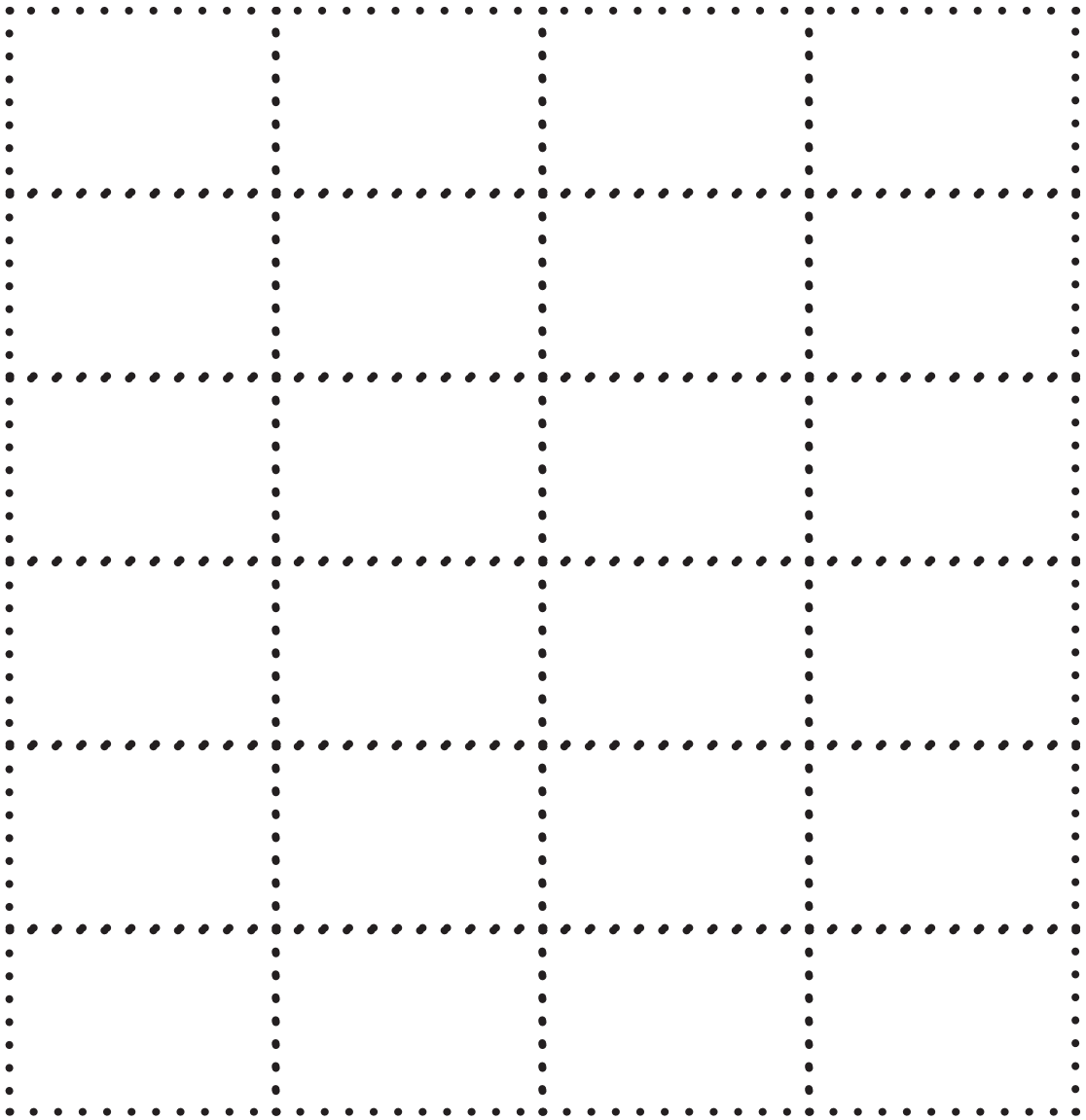
You will need 18 counters.

Only one counter can be placed in each box.

Some boxes can be left empty.

To finish the puzzle each row and each column must have an even number of counters.

Show your teacher when you complete the puzzle



Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Guess and check

RESOURCES Each pair or group of pupils will need 18 counters.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

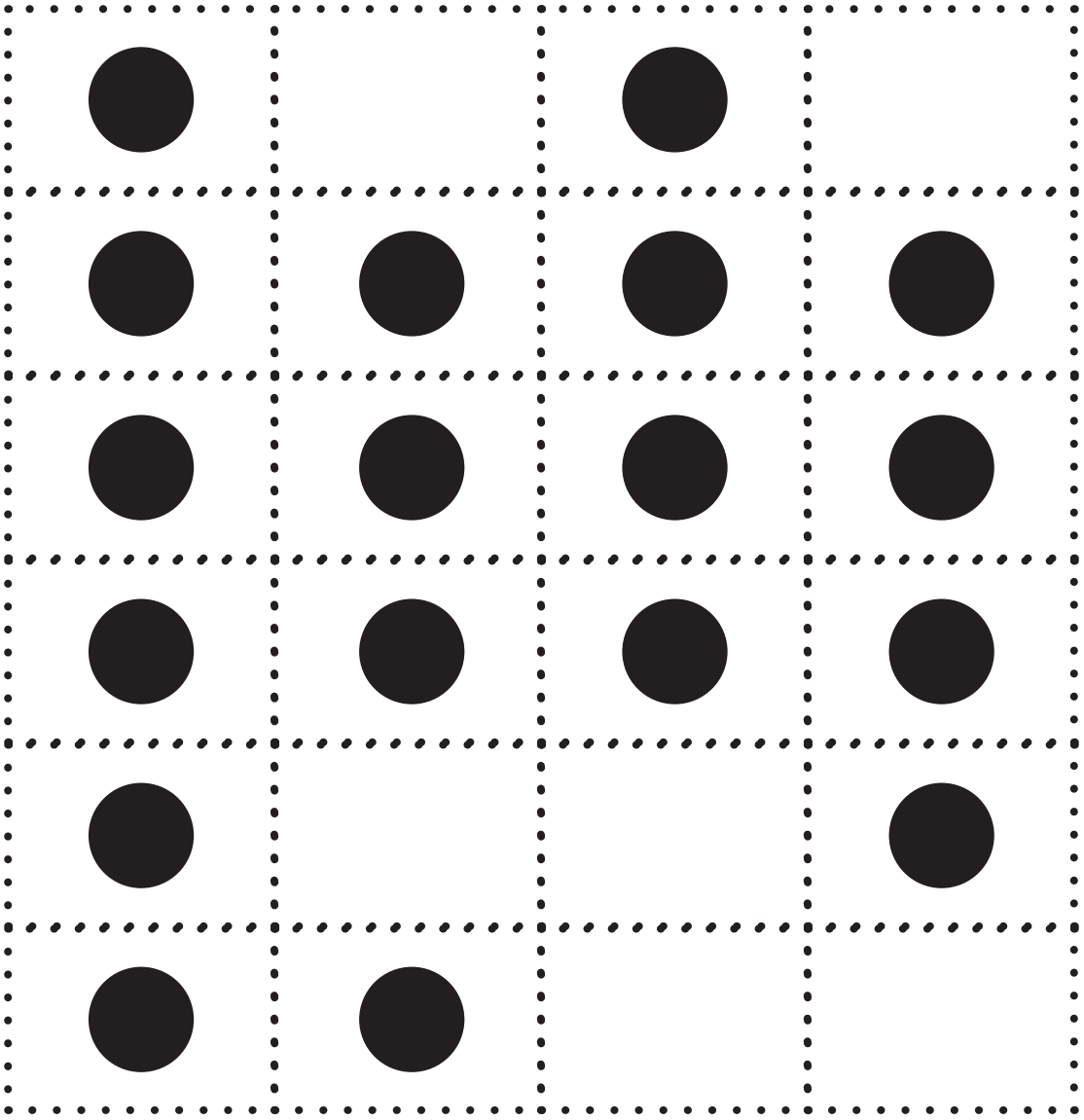
WARM UP What are the even numbers more than 11 but less than 31?

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and allow the pupils to formulate their own strategies for solving the problem.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION Bring the class together and allow several pupils to report their solutions. Discuss any difficulties.



Pizza Party

Donna is having a sleepover party. She invites Carla, Tasnim, Marita and Rosie.

For food she orders pizza for her friends.

Donna likes more than one topping on her pizza.

Carla likes ham but not pineapple.

Tasnim likes mushrooms but not ham.

Marita likes ham but not large pizzas.

Rosie likes mushrooms but not on a big pizza.

Use the worksheet to see what pizzas have arrived.

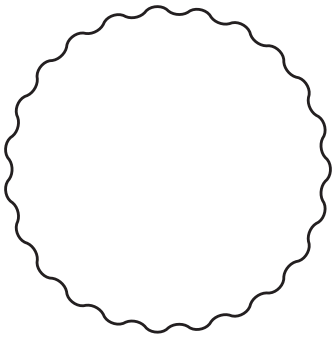
Who will eat which pizza?

Which pizza has been delivered by mistake?

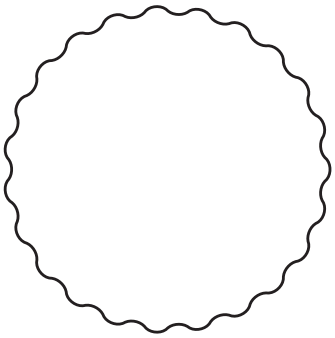
Pupil Worksheet

Pizza choices

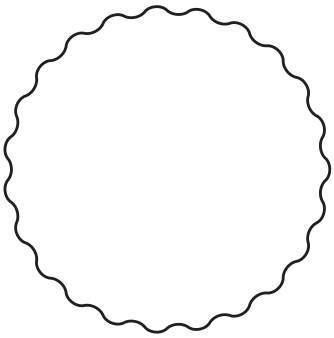
Regular



MUSHROOM & PINEAPPLE

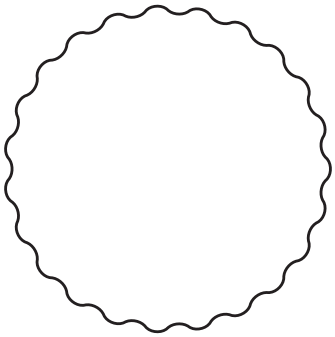


MUSHROOM

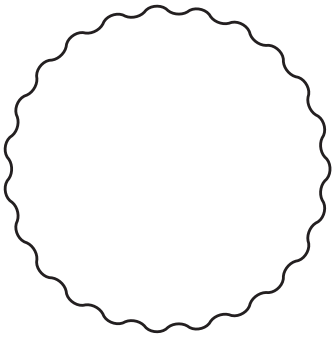


HAM & PINEAPPLE

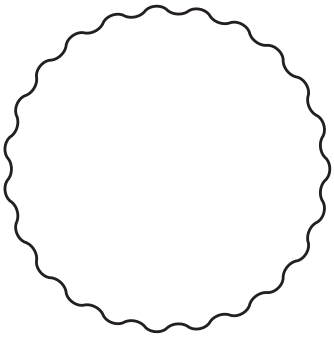
Large



PINEAPPLE



MUSHROOM & HAM



HAM

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Act it out

RESOURCES Each pair or group of pupils will need pizza cards and name cards.
Scissors will be required.

TIME 1 period

LESSON STRUCTURE a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Which is the odd one out from 5, 6, and 8?
Give a reason.
Is there more than one answer?

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
Allow pupils to cut out cards and then to act out the problem.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION Bring the class together and allow several pupils to report their solutions. Discuss any difficulties.

Donna: large mushroom and ham

Carla: large ham

Tasnim: regular mushroom and pineapple

Marita: regular ham and pineapple

Rosie: regular mushroom

Large pineapple delivered by mistake.

Section 4

Problem solving tasks and lesson plans

Level C

Contents

Teachers Notes

Resources

Each lesson has:

1. Lesson starter materials
2. Pupil worksheets
3. Teacher notes
4. Solutions

Teacher notes highlight the main strategies that could be used in the problems.

The lesson starters are designed, whenever possible, to be related to the problem that will follow by highlighting possible strategies that could be used in the lesson and also to review vocabulary used in some problems.

Overheads are included for some problems to clarify diagrams or to allow games to be played by the teacher with the class.

The order in which the problems are tackled is not important.

Class structure

It is suggested that all of these problems should be tackled with the **pupils working in pairs or small groups.** Learning to reason requires interaction with someone else.

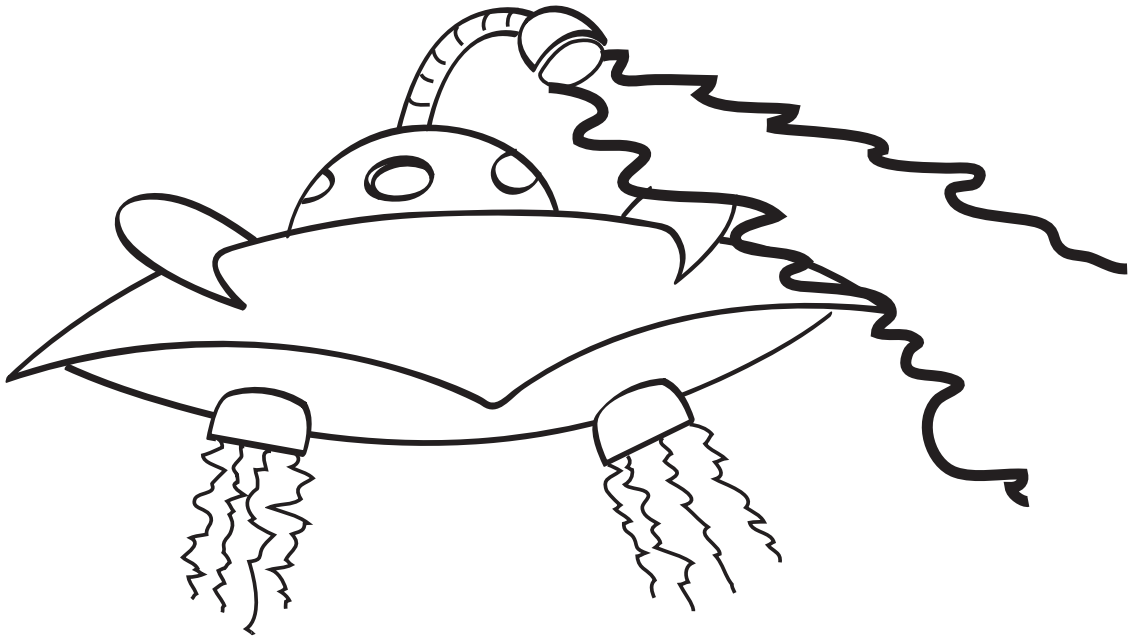
Pupils need to be challenged to think and talk about what they are doing when solving problems.

It should be stated explicitly to the class why they are in pairs or groups. It is so that they can discuss the problem and so that they can support each other. It is not just by accident that they have a partner.

Problems are best solved by pupils discussing with each other the possible strategies that can be used to tackle the problem and by supporting each other through the solving process.

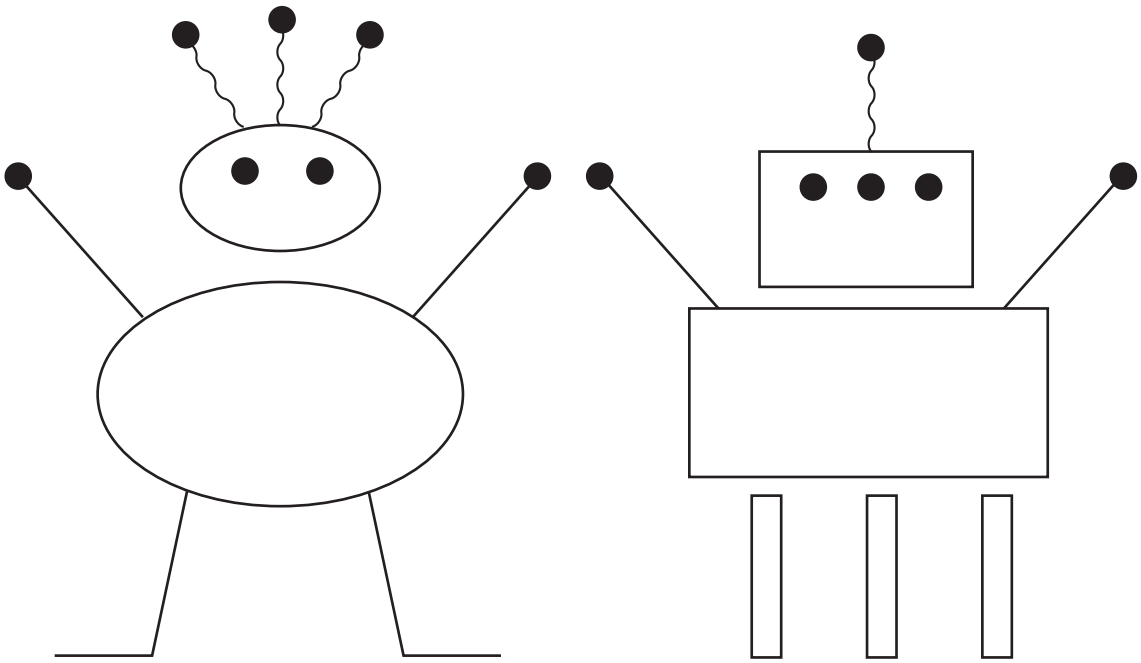
Most pupils will learn more and be more confident attempting the problems using a paired or group approach rather than asking them to solve the problems on their own.

War of the Worlds



A TV producer has asked a class of pupils to design an alien for a new science fiction programme.

One pupil designs one alien with circles and another with squares.



The pupil decides to mix up the heads, body and legs of the two aliens.

How many different combinations are possible

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Act it out
b. Organise a list

RESOURCES Card and scissors should be available, as pupils may wish to cut the aliens' head, body and legs.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP 6, 7, and 8 are three consecutive numbers which add to 21.
Can you find three consecutive numbers which add to 30?
Can you explain how you got your answer?

STARTING Read through the problem with the pupils.
Emphasise that the two types of legs cannot be combined.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
Allow pupils to draw aliens and cut them out then to act out the problem.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION Bring the class together and allow several pupils to report their solutions. Discuss any difficulties.
There are eight possibilities.

Think About It

Problem 1

Ann, Boris, Carla, and David are talking about their favourite food.
They know they all like different foods and these are:

curry	fish and chips
lasagne	chicken burgers

- Boris does not like chips.
- Ann will not eat chicken.
- One of the boys likes fish.
- One of the girls likes burgers.
- Neither of the boys likes spicy food.

Which person likes what food?

Problem 2

David, Eddie and Fred, three brothers, are marrying three sisters, Gina, Hana, and Iris.
Eddie is the oldest brother and Fred is the youngest brother. Hana is the middle sister, while Iris is the oldest.
Fred works as a plumber and Gina is a computer programmer.
From the clues below can you find out who the couples are?

- Hana is not engaged to the plumber.
- The youngest girl will marry the middle brother.

Problem 3

Make up your own puzzle and challenge your partner to find the answer.

Problem 4

A problem for the whole class.
Jack and Jill are lying dead in the middle of the room.
The only clue to what happened is some water on the floor.
What happened to Jack and Jill?
You may ask your teacher questions but they may only answer Yes or No.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Reason logically
b. Organise a list

RESOURCES None.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Countdown: Use the numbers 5, 6, 7, 8, 10, 20 to make 237.
The numbers can be used only once.
You do not have to use all the numbers.

STARTING Read through the problems with the pupils.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and allow the pupils to formulate their own strategies for solving the problem.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problems?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

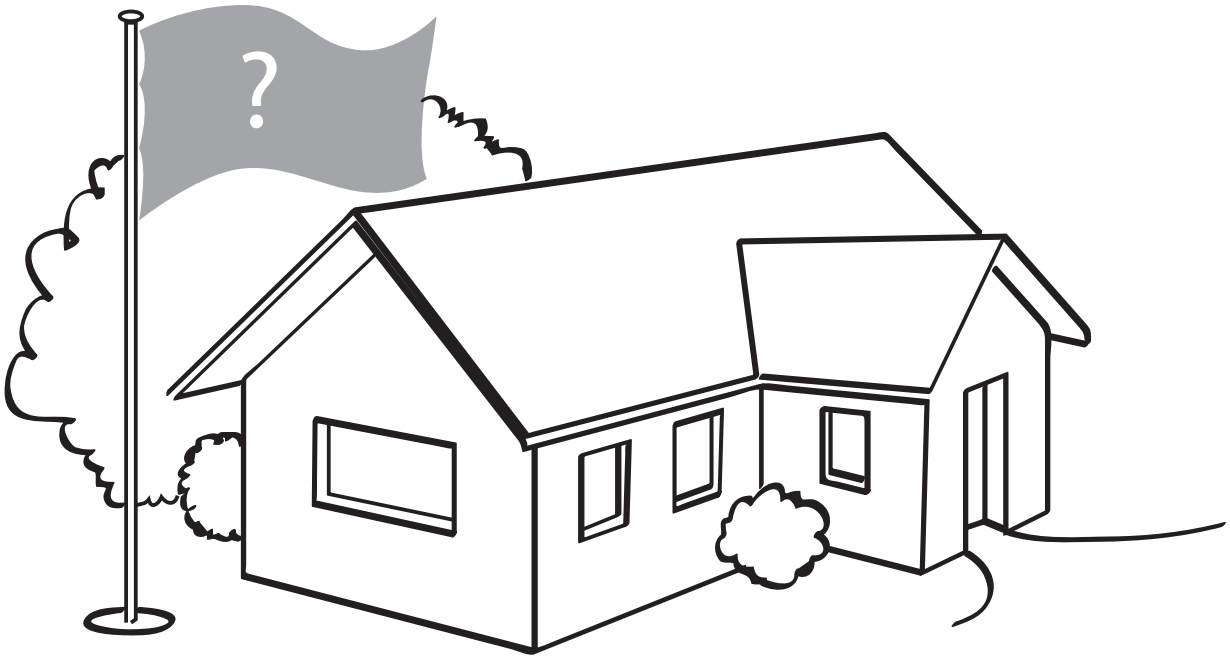
PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION **Problem 1**
Ann – curry
Boris – lasagne
Carla – chicken burger
David – fish

Problem 2
Gina – David
Hana – Eddie
Iris – Fred

Problem 4
Jack and Jill are goldfish.
The goldfish bowl has been knocked over. (The cat did it!)

Flags



Jacques and Muntar are designing a new flag for their club.

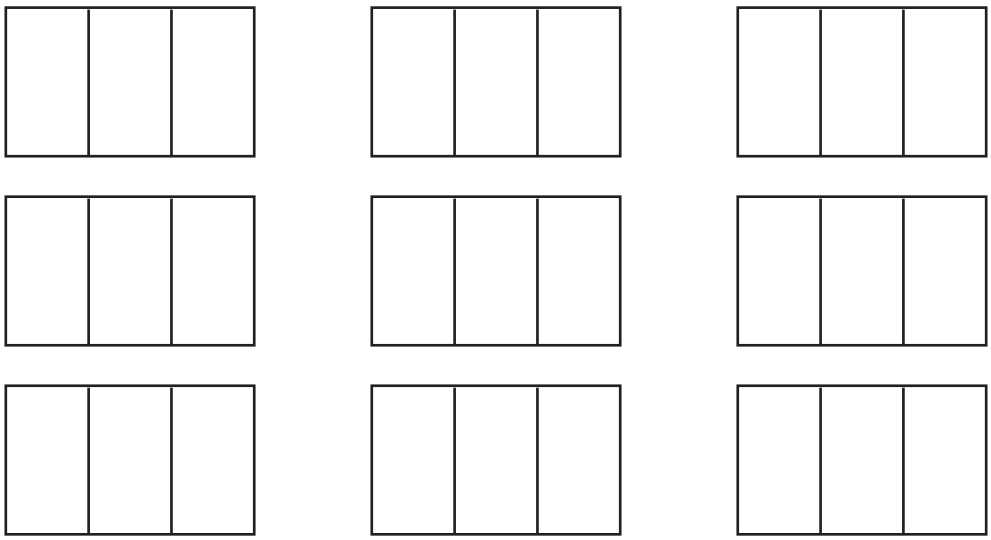
The club colours are blue and green and the flag is made up by three rectangles.

Use the worksheet to design as many different flags as possible.

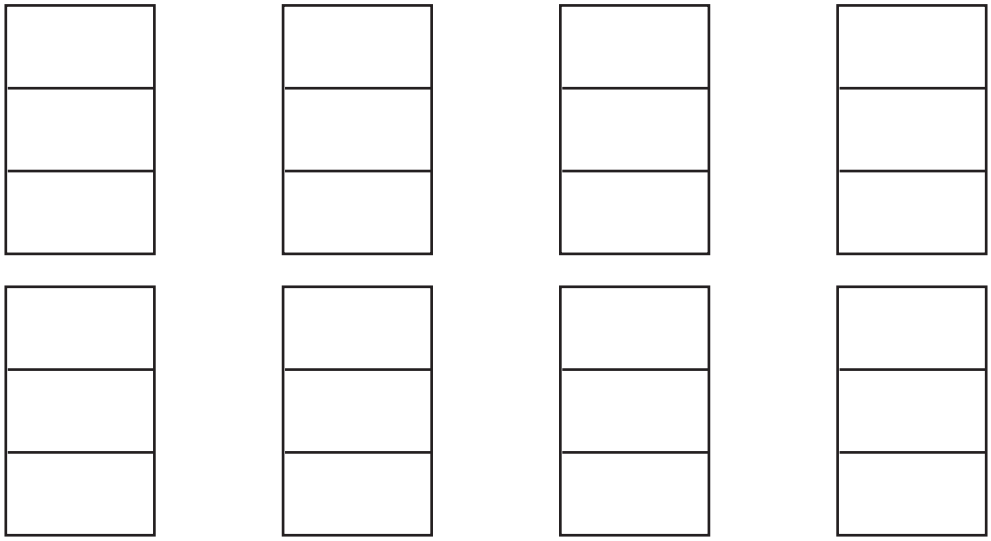
Both colours could be used or just one colour may be used.

You do not have to complete all the flags.

Pupil Worksheet

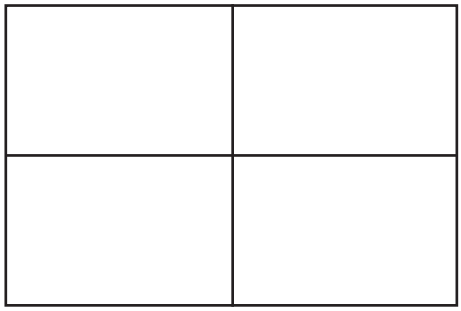


Use the colours red and yellow to colour these flags differently.



Extension

Using two colours and the flag below, how many different flags can be drawn ?



Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Organise a list

RESOURCES Coloured pencils are needed.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Countdown: Use the numbers 2, 3, 4, 5, 6, 8, 45, 50 to make 829.
The numbers can be used only once.
You do not have to use all the numbers.

STARTING Read through the problems with the pupils.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION There are 8 different flags that can be made.
The extension has 16 solutions.

Academy Awards

A game for the class

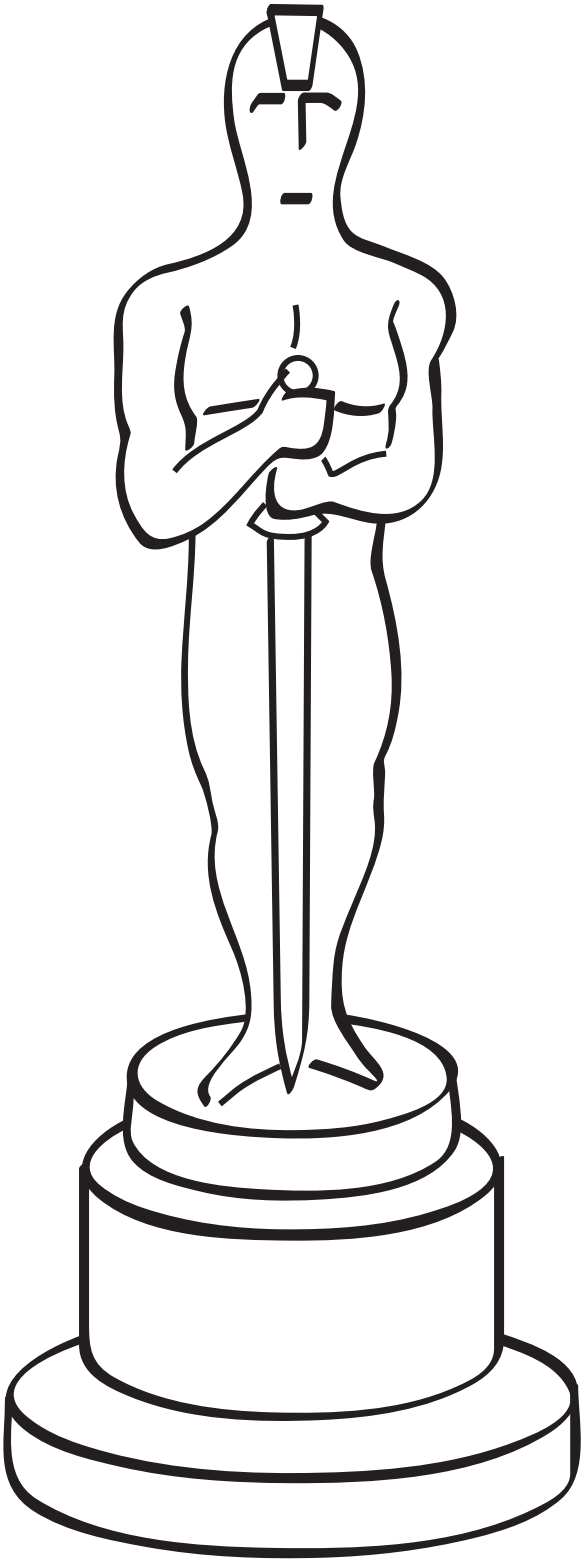
Select nine pupils from the class.

Each pupil receives a card numbered from 1 to 9.

The task for the pupils is to divide into three groups of three so that the total on the cards in each group is 15.

The game could be repeated with a different group of 9 pupils. This time the totals for each group should be different. They should be 16, 15, 14.

The game could be repeated again. This time the totals for groups could be 12, 15 and 18.



Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Act it out
b. Guess and check

RESOURCES Cards will be needed, with the numbers 1 to 9 on them.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP A game for the class
The teacher should think of a number between 1 and 1000.
Pupils have to ask questions to identify the number.
The game could be made more difficult by specifying the number of times certain questions could be asked. For example: 'Is the number larger than……?' can only be used three times.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and allow the pupils to formulate their own strategies for solving the problem.
The pupils should try to work out the solution as a large group.

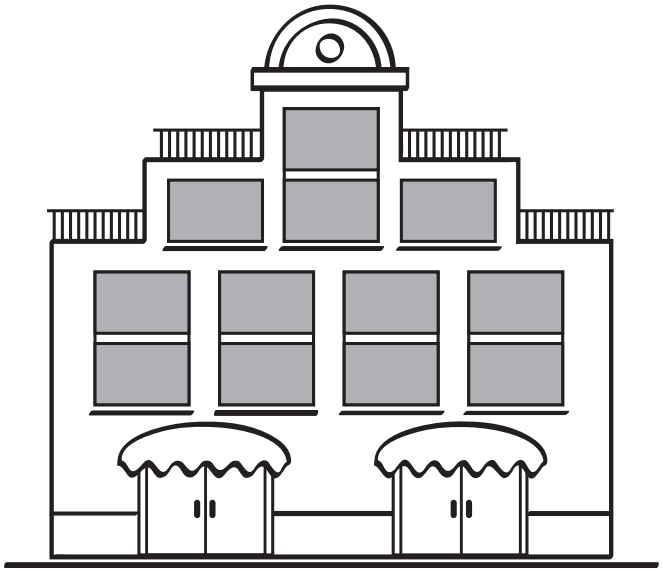
DOING Is further discussion required to support the pupils?
Ask the class who should move and why.
Repeat the process. Ask the class if there are any numbers which should not go in the same group? Why? Etc.
Are there any other methods which could help?
Would trying it on paper help?
Continue until a solution is found by the class.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION

9, 1, 5	7, 2, 6	8, 3, 4
9, 1, 4	8, 2, 6	7, 3, 5
9, 1, 2	7, 3, 5	8, 6, 4

What can you see?



Jack and Shaheen are glaziers. They have been asked to replace all the rectangular windows in the building above.

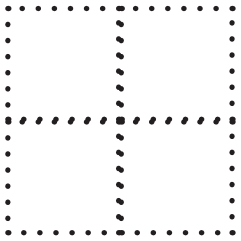
'That will not be expensive as there are only 7 rectangular windows,' says Jack.

'You can't count,' says Shaheen.

Can you explain this?

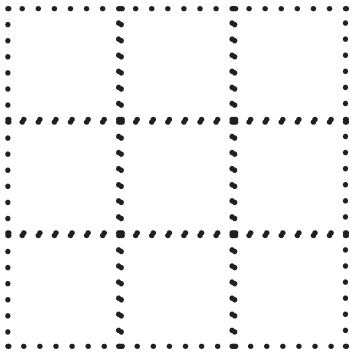
Problem 1

How many squares can you see?



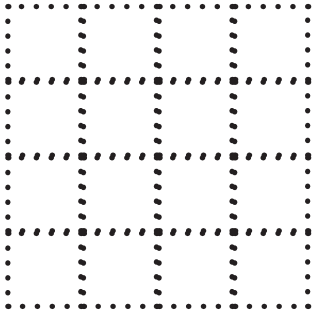
Problem 2

How many squares can you see?



Problem 3

How many squares can you see?



Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Act it out
b. Organise a list
c. Make a model

RESOURCES The problem could be assisted if the pupils made 2 by 2 and 3 by 3 grids to use as templates to help them count the number of squares.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP The initial picture question on the sheet is suggested as a warm up.

STARTING Read through the problems with the pupils.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problems?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION The warm up is to promote pupil discussion.
5 squares, 14 squares, 30 squares.
A chessboard could be used as an extension.

Section 5

Problem solving tasks and lesson plans

Level D

Contents

Teachers Notes

Resources

Each lesson has:

1. lesson starter materials
2. pupil worksheets
3. teacher notes
4. solutions.

Teacher notes highlight the main strategies that could be used in the problems.

The lesson starters are designed, whenever possible, to be related to the problem that will follow, by highlighting possible strategies that could be used in the lesson and also to review vocabulary used in some problems.

Overheads are included for some problems to clarify diagrams or to allow games to be played by the teacher with the class.

The order in which the problems are tackled is not important. However, it is suggested that Theme Park could be tackled at the end as it requires a variety of strategies to be used and it could be used prior to pupils sitting their National Assessment.

Class structure

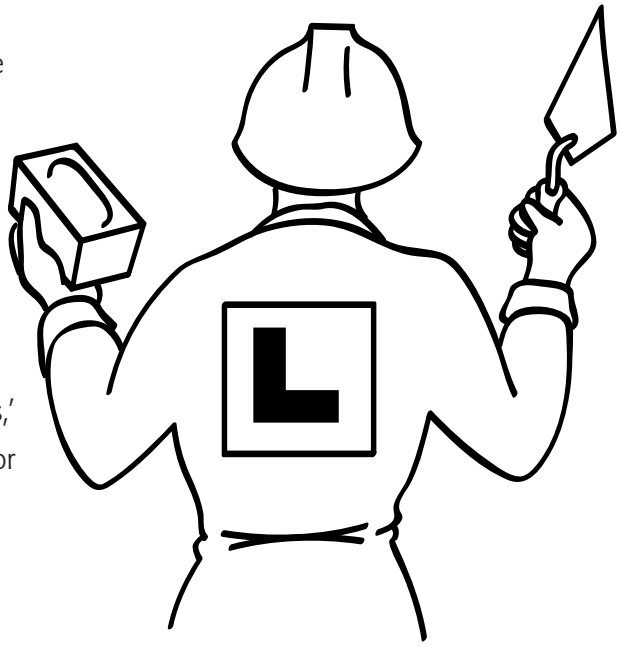
It is suggested that all of these problems should be tackled with **the pupils working in pairs or small groups.** Learning to reason requires interaction with someone else. Pupils need to be challenged to think and talk about what they are doing when solving problems.

It should be stated explicitly to the class why they are in pairs or groups. It is so that they can discuss the problem and so that they can support each other. It is not just by accident that they have a partner.

Problems are best solved by pupils discussing with each other the possible strategies that can be used to tackle the problem and by supporting each other through the solving process. Most pupils will learn more and be more confident attempting the problem using a paired or group approach rather than having to solve the problem on their own.

Apprentice Builder

A group of S4 pupils have applied to be apprentice bricklayers. The foreman asks the pupils to look at the small brick pyramid he is building and work out how many bricks are needed to build it.

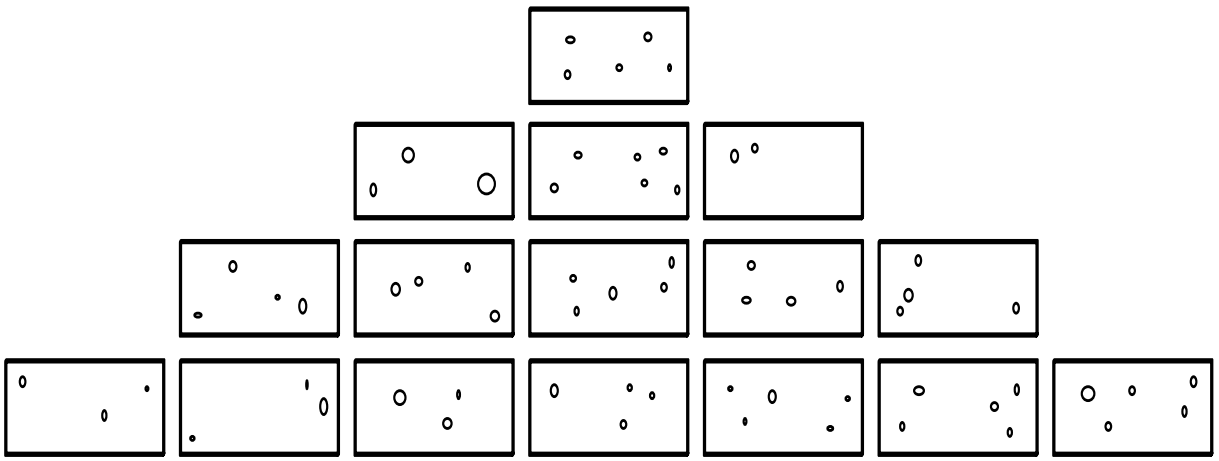


'That's easy,' says Usman. 'It is 4 steps high and the total number of bricks required is 16.'

'Excellent. But this is just a model. We will have to work out how many bricks are needed for larger walls,' replies the foreman. 'How many bricks are required for a 10-step pyramid?'

'I'm stuck. Can't we make it easier?' says Usman.

The 4-step brick pyramid



The problems

a. Look at the diagram of the 4-step high brick pyramid on the first sheet. Draw or make a 2-step high pyramid.

b. Now draw or make a 3-step high pyramid.

c. How many bricks would be required for a 2-step high pyramid?

d. How many bricks would be required for a 1-step high pyramid?

e. Copy and complete this table:

Number of steps	1	2	3	4
Number of bricks				16

f. Discuss with your partner what would the best way to find the number of bricks for a 10-step pyramid? Now use this method to try to calculate the number of bricks required.

Extension

Work out how many bricks would be required for a 20-step pyramid.
Write a rule that would allow you to work out how many bricks are required for any number of steps.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:

a. Draw a diagram
b. Looking for a pattern
c. Try a simpler case.

RESOURCES The pupils will need rulers.
Pupils have two worksheets that state the problem and outline the tasks.
4-step pyramid diagram/visual aid is included.
A warm up/lesson starter is included.

TIME 1 period

LESSON STRUCTURE

a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP The warm up encourages the pupils to look for patterns.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
It is important that the pupils are fully aware of the design of the pyramid.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, ten minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:

a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION c. A two step pyramid requires 4 bricks.

d. A 1 step pyramid requires 1 brick.

e.

Number of steps	1	2	3	4
Number of bricks	1	4	9	16

f. The problem can be solved by adding on 3 then 5 then 7, etc, or by spotting the square numbers. A 10-step pyramid requires 100 bricks.

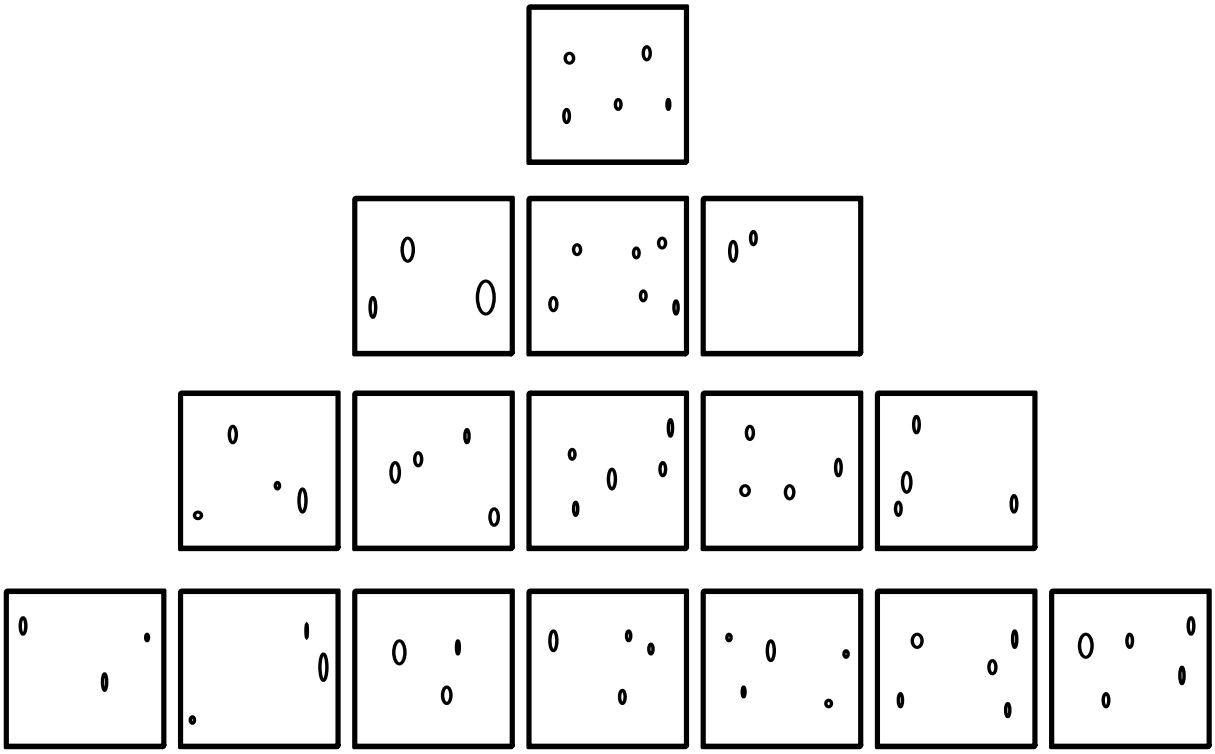
EXTENSION 20 steps = 400
For n steps, the number of bricks = n^2

Warm-up activity/Lesson starter

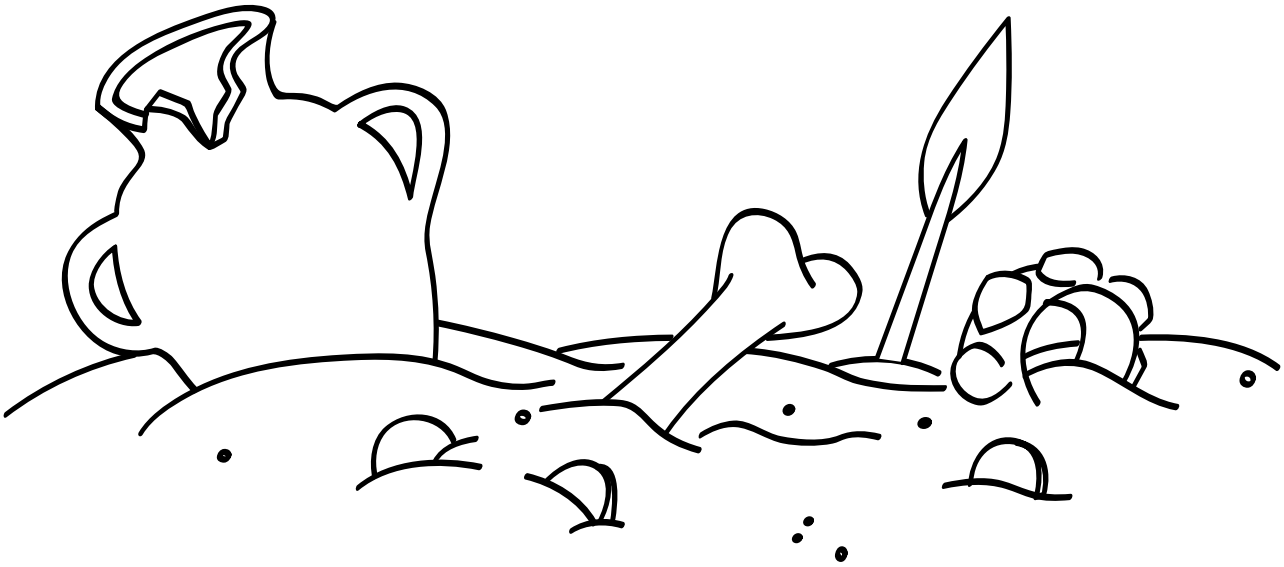
Complete the patterns

1. 4, 8, 12, 16, ?, ?
2. 5, 6, 11, 17, 28, ?, ?
3. 12, 11, 9, 6, ?, ?
4. 450, 750, 1050, ?, ?
5. 1234, 123.4, 12.4, ?, ?

The Four-step Pyramid



Archaeological Dig



As part of the S4 work experience programme Stacey and Farzad join a team of archaeologists who are digging in the grounds of Crookston Castle to look for any ancient artefacts.

Before they begin to dig they mark out the ground into a grid of squares so that they can map out where any finds are made.

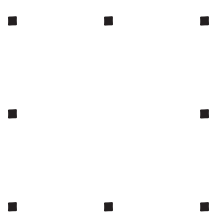
Stacey and Farzad have the task of getting the pegs for the squares.

Unfortunately they are not sure how many pegs are needed.

Use the worksheet to help Stacey and Farzad solve the problems.

The problems

a. The archaeologist asks for a square with 3 pegs on every side to be mapped out. Stacey draws a sketch and tells Farzad that they need 8 pegs.



Draw a square that has 4 pegs on every side. How many pegs are needed?

b. Draw a square that has 5 pegs on either side. How many pegs are needed?

c. The archaeologist then asks for a square that uses 28 pegs. Draw this square. How many pegs are on each side?

d. How many pegs would be on each side if the square required 36 pegs?

e. The last piece of ground is in the shape of a rectangle. Farzad and Stacey manage to map it out using 26 pegs. The longest side of the rectangle has one more peg than the shorter side. How many pegs are on each side? Draw the rectangle.

Extension

The archaeologists discover that the castle had a street with 16 houses equally spaced out in a row. They discover the names of 4 people in the street: Alanus, Borisa, Caligula, and Dana. They also discover clues as to which house each person lived in.

There are 6 houses between Borisa and Alanus.

There are 4 houses between Alanus and Caligula.

Borisa and Caligula live furthest apart.

Caligula and Dana are the closest, being next-door neighbours.

Borisa lives at number 1.

What numbers do the others live at?

Discuss with your partner the best way to do this.

What problem solving strategy would help?

Now try to solve the problem.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Draw a diagram
b. Guess and check.

RESOURCES The pupils will need rulers.
Pupils have two worksheets that state the problems and outline the tasks.
A warm up/lesson starter is included.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP The warm up encourages the pupils to look for patterns.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
It is important that the pupils are fully aware of the position of the pegs in the square.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION TThe plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain or demonstrate their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION

a. 12 pegs
b. 16 pegs
c. 8 pegs on each side of the square
d. 10 pegs on each side of the square
e. 8 pegs by 7

EXTENSION Drawing a diagram and guessing and checking would be a suitable strategy.

B = 1
A = 8
D = 12
C = 13

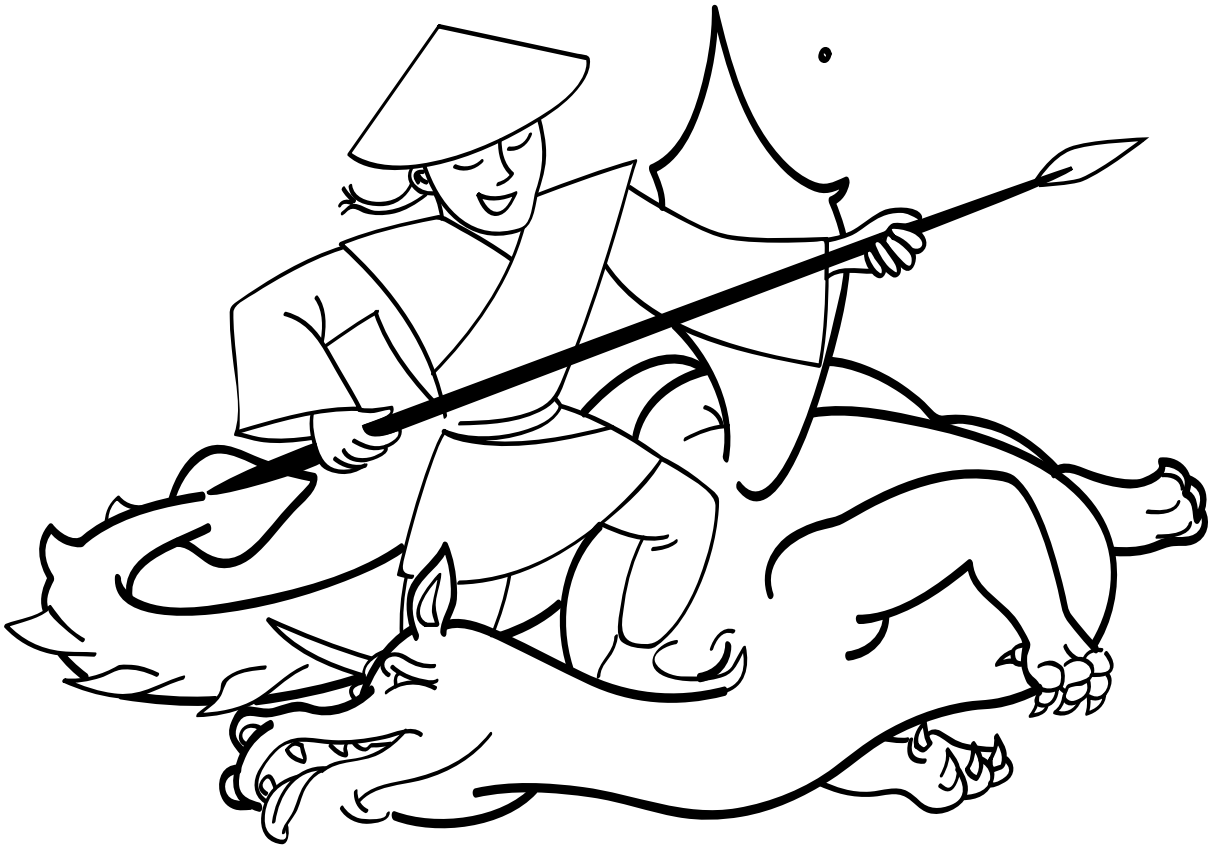
Warm-up activity/Lesson starter

Using the numbers 4, 5, 8, 6, 7, 9 at most once, write down the largest possible 4-digit number.

Write down the smallest 4-digit number, again using the numbers at most once.

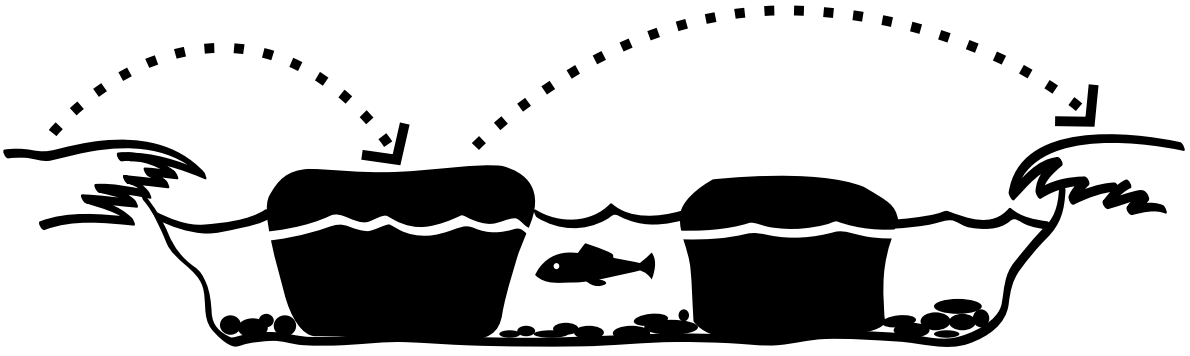
What is the difference between these two 4-digit numbers?

Barbaran the Dragon Fighter



A class of S1 pupils are designing their own computer game, Barbaran the Dragon Fighter. As part of the game Barbaran must cross several rivers by leaping from the riverbank on to stepping-stones. He can either jump to the first stone or to the second stone each jump.

One way to cross a river with 2 stepping-stones is given below.



So the river was crossed with one short leap and one long leap, or S + L.

The S1 pupils realise that there are several more ways to cross the river.

The problems

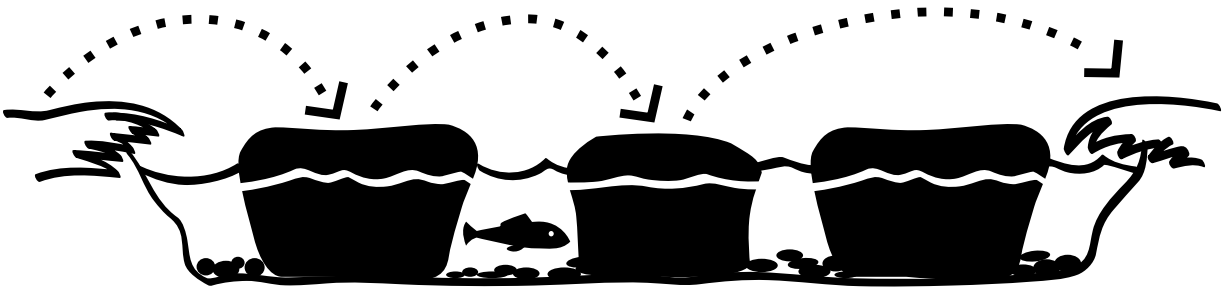
a. In your group or in pairs use your counters to find out other ways to cross the river.

b. List all the different ways to cross the river, using S for a short jump and L for a long jump.

c. The next river to be crossed by Barbaran has 3 stepping-stones. Again he can only jump with a short leap to the first stone or a longer leap to the second stone until he crosses the river.

Use counters to find out the different ways to cross the river.

One example is given below.



This is short, short, long; or S, S, L.

d. List all the different ways to cross the 3 stepping-stone river.

e. Copy and complete the table below:

Number of stones	0	1	2	3	4
Number of ways to cross	1				

f. Discuss with your partner or group what would be the best way to find out the number of different ways there are to cross a river with 8 stepping-stones.

Now try to find the number of different ways to cross when there are 8 stones.

Extension

Explain a rule that would allow you to work out how many different ways there are to cross a river with 10 stepping-stones.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Organised list
b. Looking for a pattern
c. Act it out.

RESOURCES Pupils have two worksheets that state the problem and outline the tasks.
Counters should be available to the pupils to allow them to act out the problem.
A diagram/overhead sheet is included to assist the teacher to clarify the problem.
A warm up/lesson starter is included.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP To reinforce guess and check.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
It is important that the pupils realise the meaning of **short and long jumps**.
There is an opportunity for groups of pupils **to act out the problem** or for a group of pupils to act out part of the solution **as part of the introduction to the class.**
This would clarify the problem for many pupils and allow them to achieve more.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION b. SSS, SL, LS

c. SSSS, SSL, SLS, LSS, LL

e.

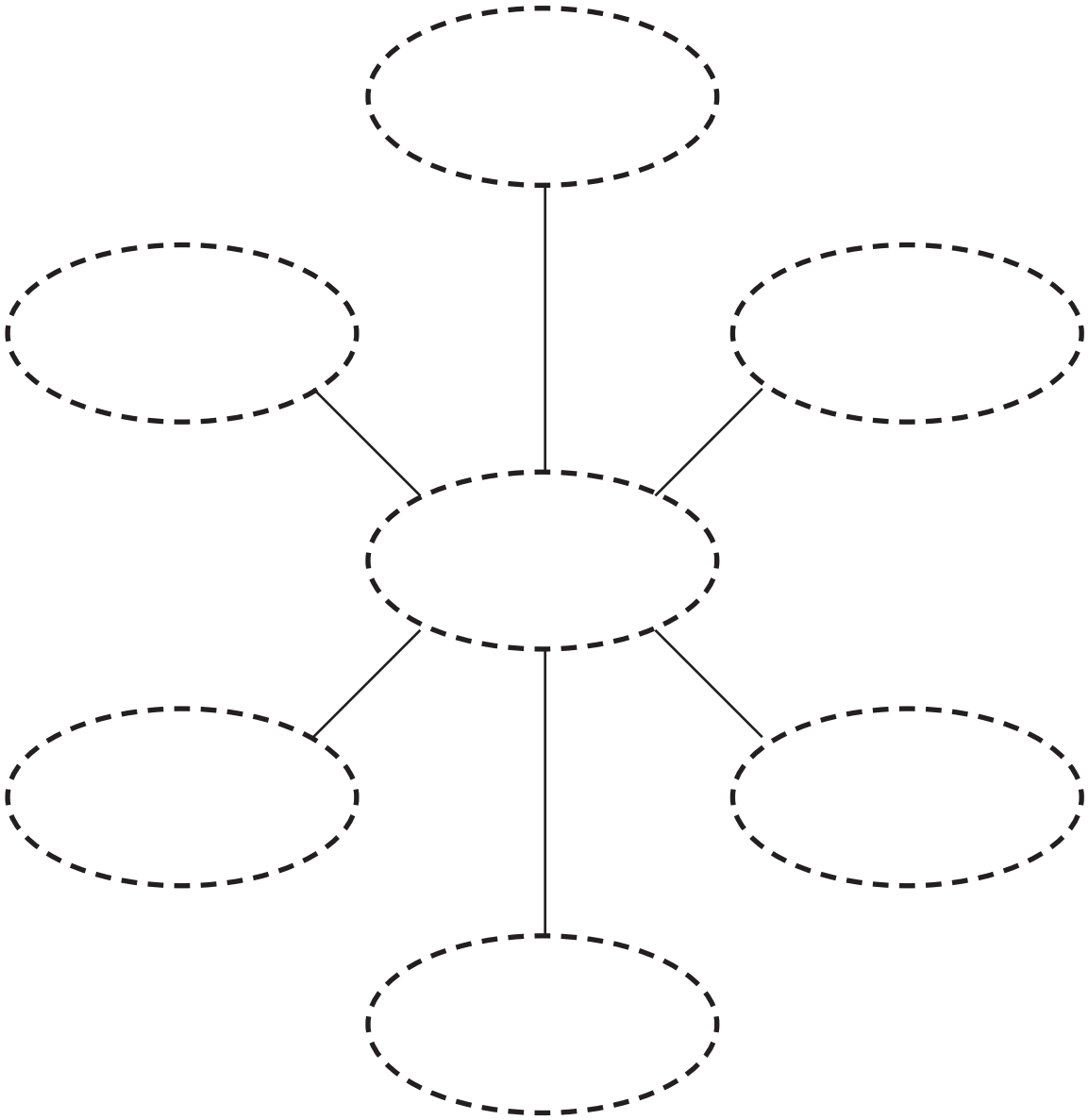
Number of stones	0	1	2	3	4
Number of ways to cross	1	2	3	5	8

f. Extending the table is one method. There are 55 ways for 8 stones.

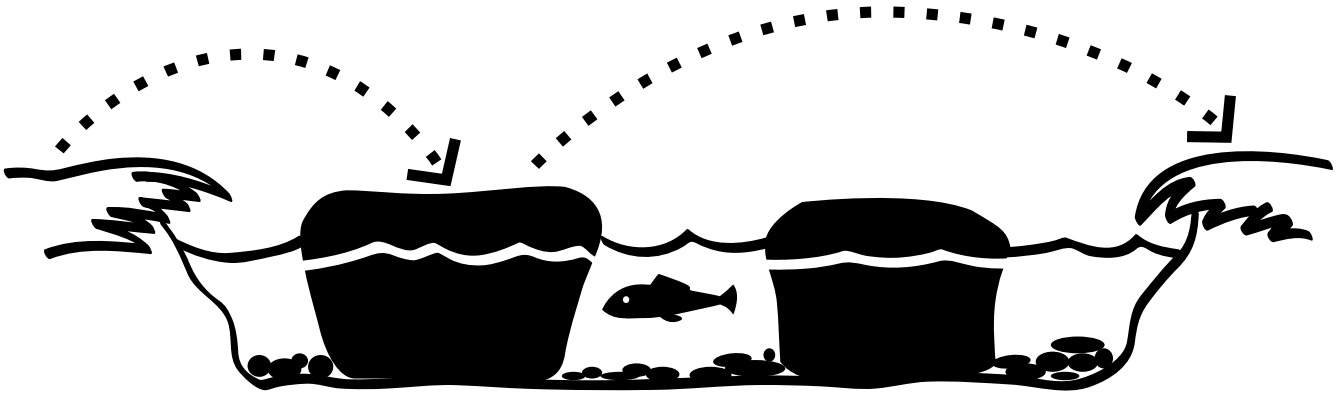
EXTENSION This is a Fibonacci sequence.
The solution can be found by adding on the previous number until the required number of stones is reached.
1 + 2 + 3 + 5 + 8 + 13…
Total is 144.

Warm-up activity/Lesson starter

Fill in the digits 1, 2, 3, 4, 5, 6, 7 so that each line has a total of 10.



This is an example of a short and long jump.



Crazy Painters

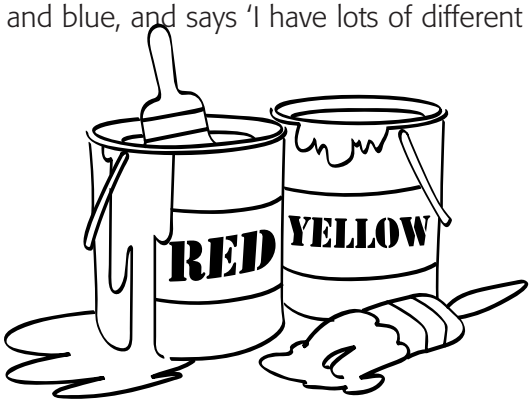
Mark, Mohammed, Martin and Maria all decide to paint one wall in their bedroom by drawing a horizontal line across the wall and so dividing the wall in two. They then go out and buy some paint.

Mark buys only two colours, red and yellow, and says 'I have a few different ways I could paint the wall.'

Mohammed buys three different colours, red and yellow and blue, and says 'I have lots of different ways I could paint the wall.'

Martin buys four different colours, red, yellow, blue, and green, and says 'I'll have to work out how many different ways I could paint the wall.'

Maria says 'I couldn't decide and bought five colours, red, yellow, blue, green and orange. I have no idea how many different ways I could paint the wall. Can you help me?'



Here are two ways Mark could paint his wall:

Red	Red
Yellow	Red

There is a worksheet on the next page, which could help you to complete the problems.

The problems

Use the worksheet to help you

a. List all the different ways that Mark could paint his wall using either red or yellow.

b. List all the different ways that Mohammed could paint his wall using red, yellow or blue.

c. List all the different ways that Martin could paint his wall using red, yellow, blue or green.

d. Copy and complete the table below:

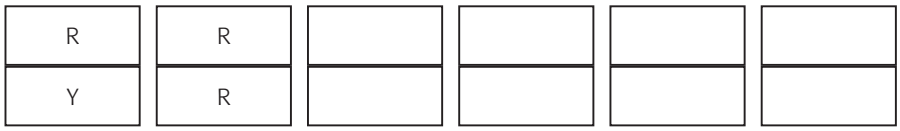
Number of colours	1	2	3	4
Number of different ways	1			

e. Discuss with your partner the best way to find out how many different ways Maria could paint her wall choosing from five colours.

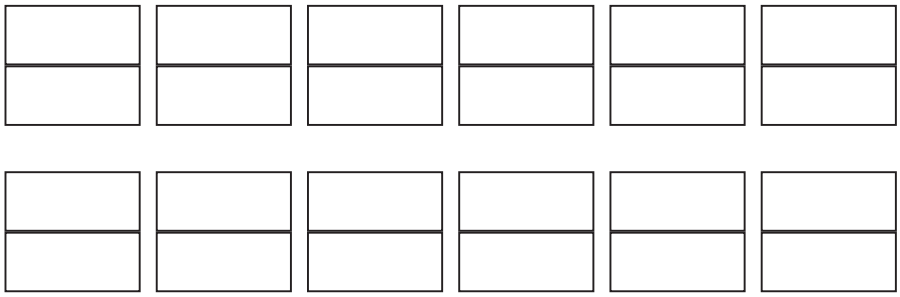
f. Help Maria by finding out many different ways she can paint her wall. Show your working.

Use the worksheet below to help you complete the problems. There are more rectangles than are required for each question.

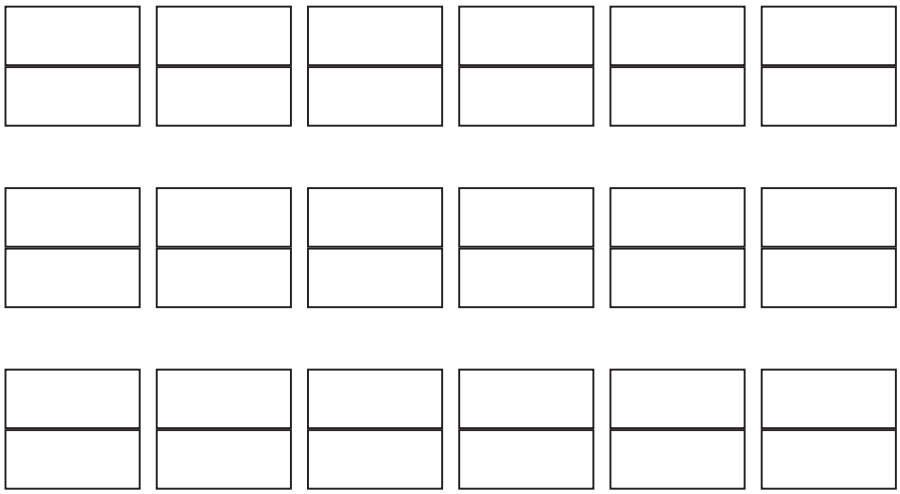
a.



b.



c.



Extension

Work out how many different ways there are to paint the wall if you had 10 colours.

Can you write a rule that would allow you to work out how many different ways there are for any number of paints?

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
Organised list
Try a simpler case
Looking for a pattern.

RESOURCES Pupils have worksheets that state the problem and describe the tasks.
Colour pencils are optional.
A warm up/lesson starter is included.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP The warm up encourages the pupils to look for patterns.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
It is important that the pupils are aware that the same colour can be used to paint both parts of the wall.
Some pupils would benefit from the use of colour pencils.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION a. RR, RY, YR, YY

b. RR, RY, RB, YY, YR, YB, BB, BY, BR

c. RR, RY, RB, RG, YY, YR, YB, YG, BB, BY, BR, BG, GG, GB, GY, GR

d.

Number of colours	1	2	3	4
Number of different ways	1	4	9	16

e. By following the pattern or by extending the table or by listing Maria has 25 ways to paint the wall.

EXTENSION For 10 colours there are 100 different ways.
The rule could be described in terms of adding from the number before, or if n colours, then number of different ways to paint the wall = n^2.

Warm-up activity/Lesson starter

Write down the next two numbers in these number patterns:

1. 2, 4, 6, 8…

2. 7, 14, 21, 28…

3. 100, 85, 70, 55…

4. 1, 4, 9, 16…

5. 2, 3, 5, 8, 13…

Friday the 13th

On period 6 on a Friday the 13th, an S1 maths class decide to play a game for 8 pupils. The pupils stand in a circle and count round the circle clockwise. The first pupil shouts one, the next pupil shouts two, the third pupil three and so on until thirteen is reached. The thirteenth pupil is out and goes back to their seat. The count then starts again at the next pupil, they shout one, the next two, and so on until the pupil who shouts thirteen is again eliminated and sits down. Eventually only one pupil is left and they are the winner.

Craig says 'I'm lucky. I could win the game each time.'
To the class's amazement Craig wins the next 3 games.
'You can't be lucky all the time,' says Farza.
'I can if I want to be,' says Craig.
'I don't understand,' says the confused maths teacher.

Play the game for 8 pupils.

Make sure you understand the rules before you start.
Stand in a circle and count clockwise.
Do not change your position after the game has started.
Remember the thirteenth person sits down, and the person after that starts again at 1.

The problems

Use the worksheet attached to help you answer the problems.

a. Choose a starting point and play the game by crossing out the person eliminated each time. Remember to start from the person after the person eliminated when counting on.
b. Write who won the game.
c. Start the game at Farza, i.e. she would say 1. Who wins?
d. Now start at Craig. Who wins the game this time?
e. Now start at Eddie. Who wins the game this time?
f. Talk to your partner. What have they found out about the game? Tell them how you think Craig fixed the game so that he won the game each time.
g. Explain in a few sentences what you would do to make sure that you would win the game.

Extension

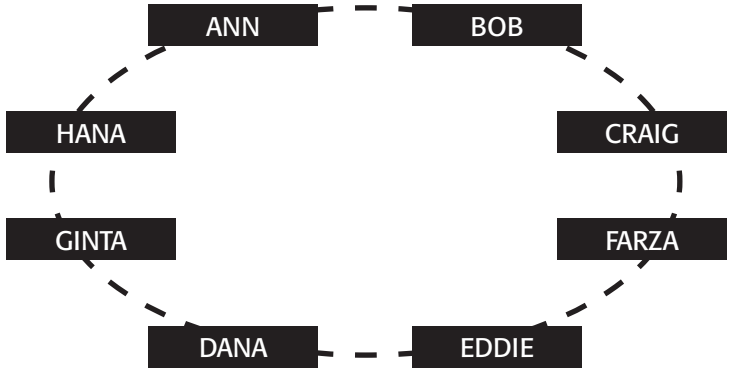
Make up a similar game and see if you can find a way to predict who will win.

It could be that every tenth person is eliminated.

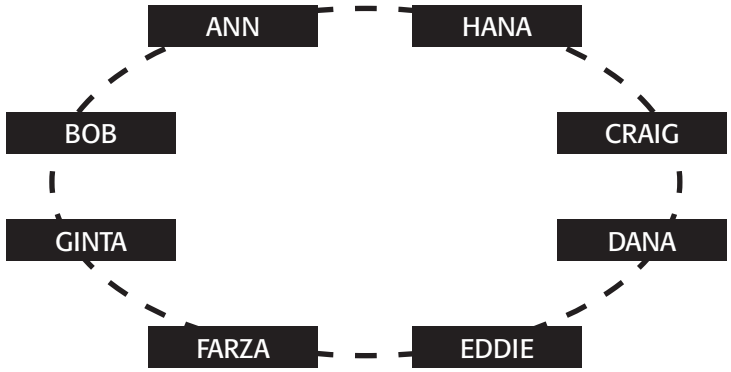
Let your partner play the game.

Pupil Worksheet

a.



b.



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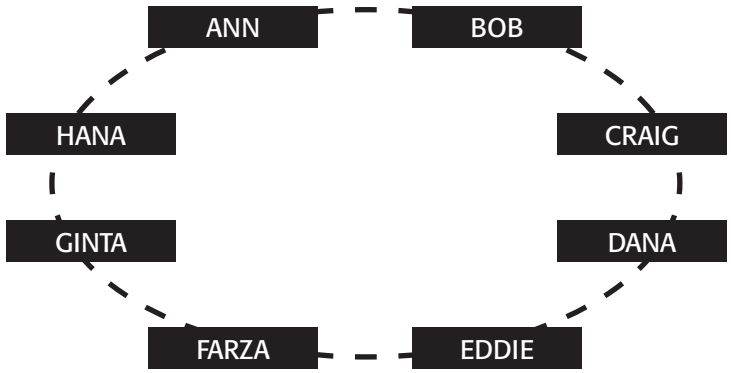
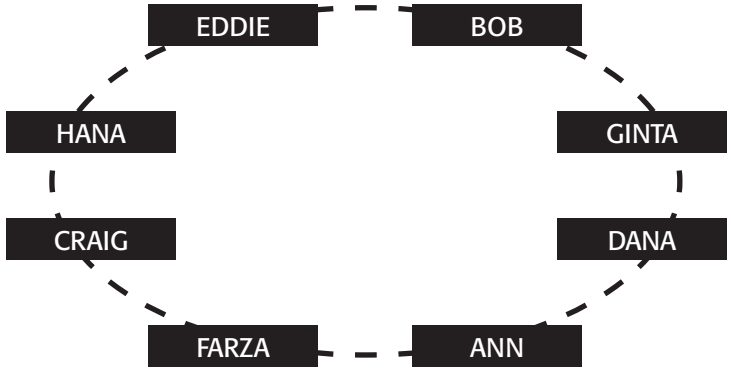


d.



Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
Act it out
Reason logically
Guess and check.

RESOURCES Pupils have two worksheets that state the problem and outline the tasks.
A warm up/lesson starter is included.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP To promote discussion on the similarities and differences of the numbers.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
It is important that the pupils understand clockwise and elimination and understand how to restart the game after a pupil has been eliminated from the game.
It may help if a ninth pupil oversees the game and the participants have labels, e.g. A to H, so that the pupils are aware of who starts and who wins.
The game should be played several times.
The game should also be played again at the end to check pupils' ideas.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?
The game may be played again to check results.

SOLUTION c. Ginta wins
d. Ann wins
e. Craig wins
Craig would win each game by standing two places to the right of the person who starts.

Warm-up activity/Lesson starter

From the three numbers below say which number is the odd one out.

7, 8, 9

There is more than one answer.

Inside Out

Worksheet 1

Find the numbers that would make the problems correct.

The problems

a. ☐ + 135 = 240

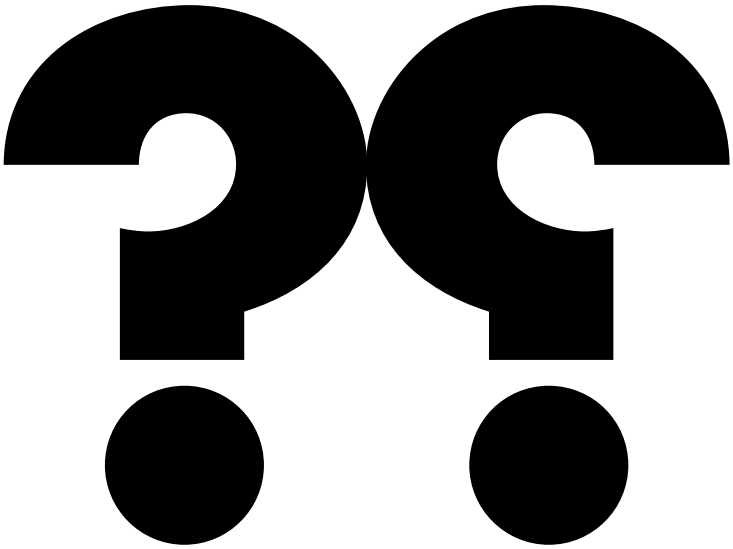
b. ☐ + 1.2 = 6.9

c. ☐ − 123 = 87

d. ☐ − 23.6 = 6.9

e. ☐ x 7 = 154

f. ☐ ÷ 20 = 30



Worksheet 2

Find numbers which would make the puzzles true.

1. If you add 5 to the number and multiply the answer by 3, you get 36.

2. Double the number then treble it and the answer is 36.

3. Half the number then add 6 and the answer is 20.

4. Half the number then add 10 and you have the number you started with.

5. Add 7 then double it then subtract 2, and the answer is 22.

6. Multiply by 5, subtract 3 and divide by 8, and the answer is 4.

7. Multiply by 5, subtract 6 and divide by 2, and the answer is 17.

8. Choose any number, multiply it by 2, then multiply by 10, then divide by 100 and then multiply by 5. What answer do you get?

9. Tom told a few of his friends his email address. Tom and all of his friends then told one more person. Everyone who knew the email address then told two more people. In total 30 people now know. How many friends did Tom tell at the start?

10. Make up your own puzzle.

Check that it works.

Now ask your partner to solve it.

Teachers Notes

AIMS To use the working backwards strategy to solve problems.

RESOURCES Pupils have two worksheets that state the problem and outline the tasks.
A warm up/lesson starter is included.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce the problems
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problems attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP The warm-up activity uses a variety of problem solving strategies.
This may require some time to complete, but the problems are shorter than usual.

STARTING Read through the problems with the pupils.
Ask them to express the problems in their own words.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with each of the problems for a short time, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle similar problems again, which strategies would they use?
Could they improve on their strategies?

SOLUTIONS

Worksheet 1

a. 105
b. 5.7
c. 210
d. 30.5
e. 22
f. 600

Worksheet 2

1. 7
2. 6
3. 28
4. 20
5. 5
6. 7
7. 8
8. The number you started with
9. Tom told 4 friends his email address at the start.

Warm-up activity/Lesson starter

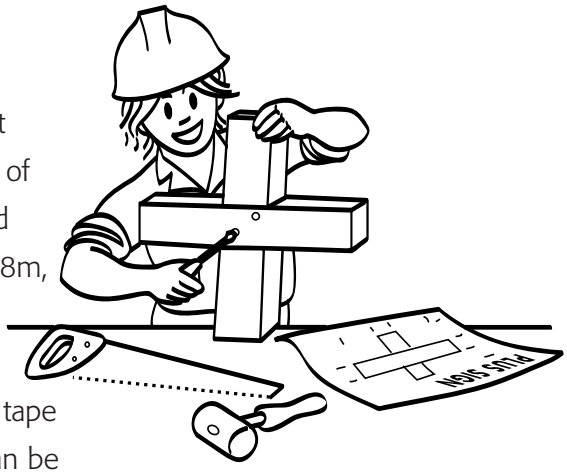
1 $666 + 66 + 6 + 6 + 6 = 750$

2 Can you use eight 8s to make 1,000 using only addition?

3 In a class of 30 there are 6 more girls than boys. How many boys are in the class?

4 I have the same number of 10p and 50p coins. Their total value is £4.20.
How many of each do I have?

Joined up Joiner



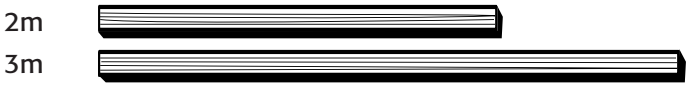
Joanne, the joined up joiner, is helping build a new house. Part of her job is to cut the wood for the roof. The roof is made up of different size pieces of wood. The length of the pieces of wood required to build the roof are 1m, 2m, 3m, 4m, 5m, 6m, 7m, 8m, 9m, and 10m.

On the Monday she cut the 2m, 3m and 7m pieces.

Unfortunately on Tuesday Joanne has forgotten her measuring tape but still has to cut the other pieces of wood so that the roof can be built.

Joanne realises that by using just the 2m, 3m and 7m pieces of wood she can still measure all the other wood sizes required to build the house.

After some thought, she has laid out the 2m and 3m pieces and used them to cut a 1m length of wood.



Help Joanne to measure all the pieces of wood using only the 2m, 3m and 7m pieces of wood.

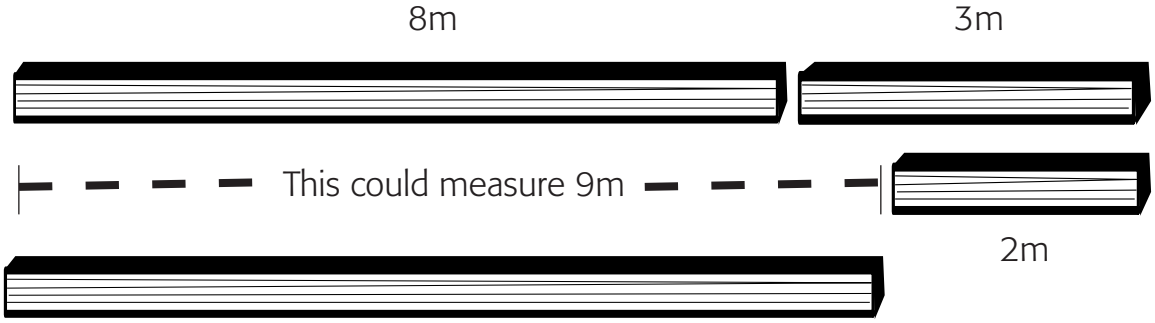
The problems

With a partner:

a. Measure, using a ruler, and cut three pieces of cardboard. Make the first 2cm, the second 3cm, and the third 7cm. Label all the pieces with the sizes so that you do not get confused.
b. **Now give the rulers back to the teacher.** Use the pieces of cardboard to measure another piece of card 10cm long and cut it out. (You do not have to use all the three pieces.) Label the piece of card 10cm.
c. Now do the same for 9cm. (Remember you can only use the 2, 3 or 7cm pieces.)
d. Talk to your partner about how you could measure 8cm using only the 2cm, 3cm and 7cm pieces. Now measure and cut out the 8cm piece.
e. Talk to your partner about how you could measure 6cm using only the 2cm, 3cm and 7cm pieces. Now measure and cut out the 6cm piece.
f. Finish the task by doing the same for 5cm, 4cm and 1cm.
g. Check that you now have 10 pieces of cardboard measuring from 1cm to 10cm.

Extension

If Joanne had cut 2m, 3m and 8m pieces of wood on the Monday, list – giving examples – all the other sizes it is possible to measure using the 2, 3, and 8m pieces. The first one is done for you.



Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Act it out
b. Make a model.

RESOURCES Scissors, rulers, cardboard or straws are required.
Pupils have two worksheets that state the problem and outline the tasks.
A warm up/lesson starter overhead projector sheet is included.

TIME 1 period

LESSON STRUCTURE a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP A short problem solving task and a reminder of BODMAS.
Also allows pupils to engage in manipulating digits to create new numbers.
How many different numbers can the class come up with in 5 minutes?

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
The problem is designed to be solved in kinaesthetically.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Hand out rulers to enable the 2, 3 and 7cm pieces to be made.
When this task is completed **remove the rulers** from the pupils and allow them to continue.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?
Could they tackle the problem without making a model?

SOLUTION

b. $10 = 7 + 3 + 2$
c. $9 = 7 + 2$
d. $8 = 7 + 3 - 2$
e. $6 = 7 + 2 - 3$
f. $5 = 3 + 2$
$4 = 7 - 3$
$1 = 3 - 2$

EXTENSION

$1 = 3 - 2$
$5 = 3 + 2$
$6 = 8 - 2$
$9 = 8 + 3 - 2$
$10 = 8 + 2$
$11 = 8 + 3$
$13 = 8 + 2 + 3$

Warm-up activity/Lesson starter

Use the digits 4, 4, 4, 4 and the symbols +, –, x, ÷ to make as many whole numbers as possible.

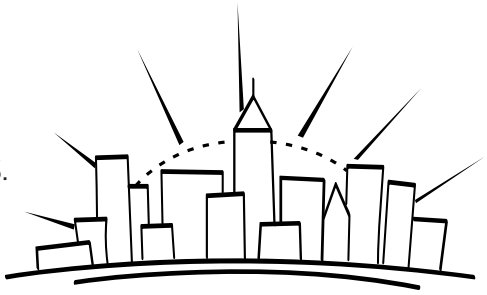
All the digits and all the symbols do not have to be used but you cannot use the symbols more than once.

Examples:

4 + 4 = 8

4 ÷ 4 + 4 x 4 = 1 + 16 =17

New Town

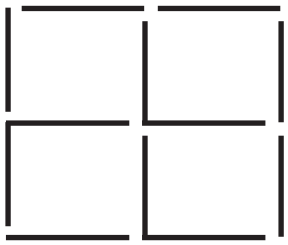


Hamid and Rachel are playing a computer game called BIMS.
The game requires them to design and build a new town.
They must include the design for the inside of the buildings.
The next building they must design is in the shape of a square and must have 100 rooms.
Unfortunately they only have enough building materials to build 230 walls.
Hamid draws one room and says 'One room needs 4 walls. So 100 rooms must need 400 walls. We only have enough materials for 230 walls. We have lost the game.'
Rachel draws some more rooms and says 'I think we have enough to build the square with 100 rooms. We can win.'
Use the worksheet on the next page to help out Rachel and Hamid.
Will they win the game?

The problems

a. Here is a 1 by 1 square.
Four walls are required.

Look at the diagram of the 2 by 2 square.
This design requires 12 walls.



b. Draw a 3 by 3 square.
c. How many walls would be required for the 3 by 3 square?
d. Copy and complete this table:

Size of Square	0 by 0	1 by 1	2 by 2	3 by 3	4 by 4
Number of walls	0	4			

e. Discuss with your partner what would be the best way to find the number of walls for a 10 by 10 square?
f. Now use this method to try to calculate the number of bricks required.
g. Rachel and Hamid have enough materials to build 230 walls. Do they have enough materials to build the 10 by 10 square and so win the game?

Extension

Work out how many walls would be required for a 20 by 20 square.

Write a rule that would allow you to work out how many walls are required for any size of square.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:

a. Draw a diagram
b. Looking for a pattern
c. Try a simpler case.

RESOURCES The pupils will require rulers.
Pupils have two worksheets that state the problem and outline the tasks.
A warm up/lesson starter is included.

TIME 1 period

LESSON STRUCTURE

a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP The warm up encourages the pupils to look for patterns within a 2 by 2 square.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
It is important that the pupils are fully aware of the design of the building.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:

a. Work systematically
b. Record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION c. A 3 by 3 requires 24 walls.

d.

Size of Square	0 by 0	1 by 1	2 by 2	3 by 3	4 by 4
Number of walls	0	4	12	24	40

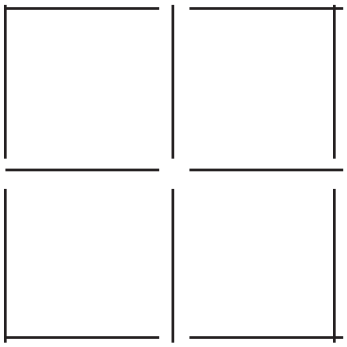
e. The problem can be solved by adding on 8 then 12 then 16, etc.
A 10 by 10 square requires 220 walls.

f. They have enough to win the game.

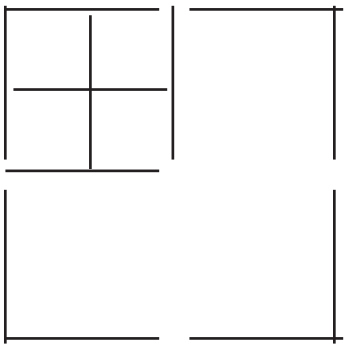
EXTENSION $2(n)(n + 1)$
20 by 20 square = 840

Warm-up activity/Lesson starter

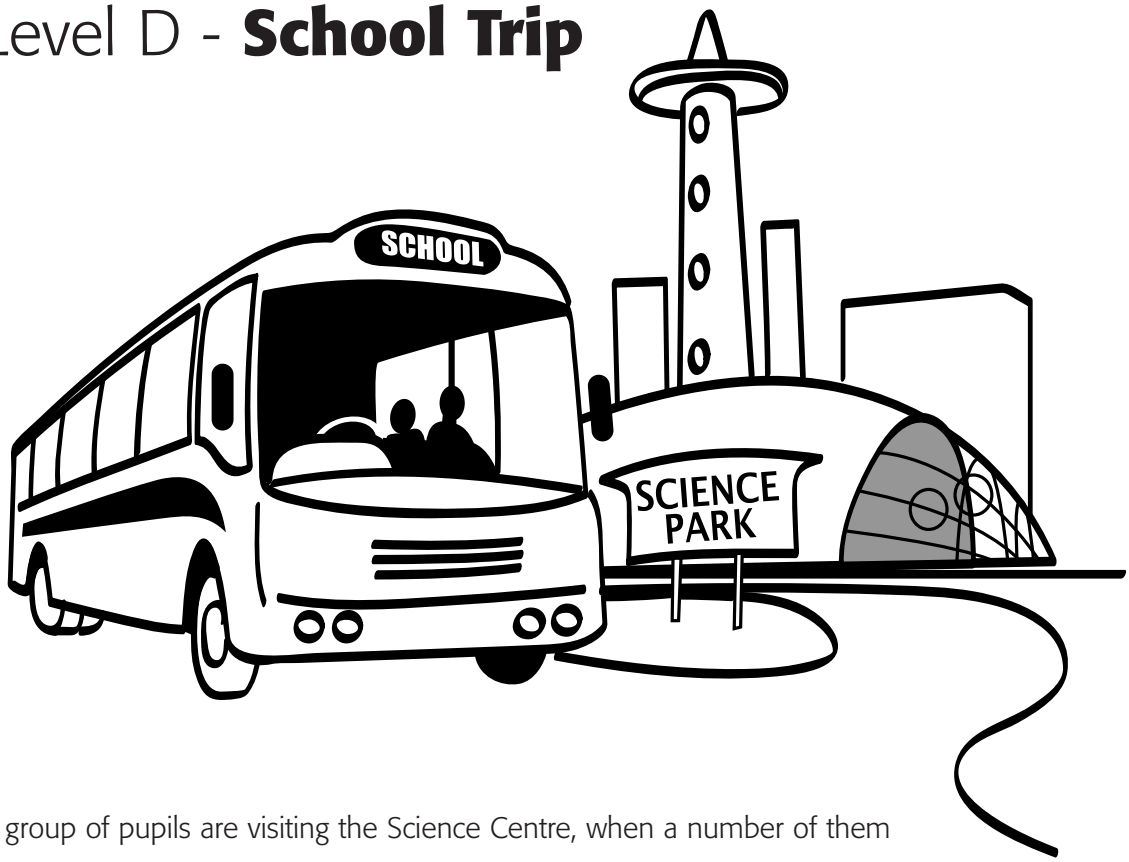
This is a 2 by 2 square.
Can you move 2 matches to make 6 squares?



SOLUTION



Level D - **School Trip**



A group of pupils are visiting the Science Centre, when a number of them decide that they want to go up to the top of the tower.
Unfortunately only two lifts are working and the teacher says 'That's fine, let's use both the lifts.'

The problems

a. List all the different ways 7 pupils can use both lifts to get to the top of tower. Remember at least one pupil must go in each lift.
b. List all the different ways 8 pupils can use both lifts to get to the top of tower.
c. List all the different ways 10 pupils can use both lifts to get to the top of tower.
d. Work out how many ways 20 pupils can use both lifts to get to the top of tower.
e. Write a rule for working out how many different ways pupils can use both lifts to go to the top of the tower.

Extension

Three lifts are now working at the tower. How many different ways can 5 pupils, 6 pupils, and 7 pupils go to the top of the tower?

Remember at least one pupil must go in each lift.

Can you write a rule for pupils using three lifts to get to the top?

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Organised list
b. Looking for a pattern
c. Act it out.

RESOURCES Pupils have a worksheet that states the problem and outlines the tasks.
A warm up/lesson starter is included.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP The warm up allows the pupils to look for patterns.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
It is important that the pupils realise that **both lifts** should be occupied **by at least one pupil.**
There is an opportunity for groups of pupils **to act out the problem** or for a group of pupils to act out part of the solution **as part of the introduction.**
This would clarify the problem for many pupils and allow them to achieve more.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.
c. use a diagram that would help to record the results.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION

Lift A	Lift B	Lift A	Lift B	Lift A	Lift B
1	6	1	7	1	9
2	5	2	6	2	8
3	4	3	5	3	7
4	3	4	4	4	6
5	2	5	3	5	5
6	1	6	2	6	4
		7	1	7	3
				8	2
				9	1

For 7 pupils there are six ways. For 8 pupils there are 7 ways.
For 10 pupils there are 9 ways.

EXTENSION

Lift A	Lift B	Lift C	Lift A	Lift B	Lift C
1	1	3	1	1	4
1	2	2	1	2	3
1	3	1	1	3	2
2	1	2	1	4	1
2	2	1	2	1	3
3	1	1	2	2	2
			2	3	1
			3	2	1
			3	1	2
			4	1	1

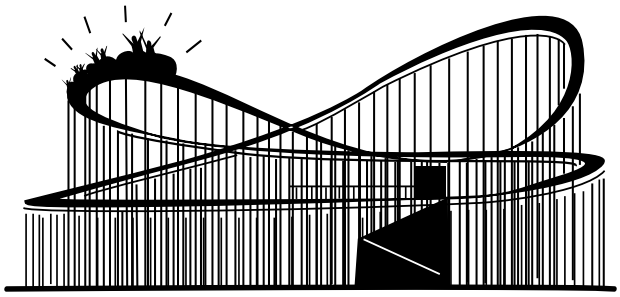
Total for 5 pupils = 3 + 2 + 1 = 6 ways
Total for 6 pupils = 4 + 3 + 2 + 1 = 10 ways
Total for 7 pupils = 5 + 4 +3 +2 + 1 = 15 ways
Rule is $(n - 2) + (n - 1) + \ldots + 1$ (n is no. of pupils)
The solution to the lesson starter is 13.

Warm-up activity/Lesson starter

How many triangles are there in the shape below?

Theme Park

The end-of-year school trip is to the local theme park. The pupils encounter various problems during the trip.
Work with a partner to try to solve the problems for the pupils.



The problems

a. The S6 pupils are in charge of ordering the buses, to go to the park, for the 152 pupils. A coach can take a maximum of 40 pupils and costs £100, and a minibus can take a maximum of 16 pupils and costs £45.
   For example, 4 coaches = 4 x £100 = £400.
   What is the cheapest way to get the 152 pupils to the park?

b. On the coach, the front double seat is empty. Amon, Brian, Carol and Dinam all want to sit in the front seat.
   List all the different ways that two pupils can sit together in the front seat.
   For example, Amon with Brian or Brian with Amon (this is not the same: why?)

c. Khalid, Angela and Stephen try to get on the Avalanche ride but the attendant says to one of them 'Sorry, you are too small.'
   Khalid is taller than Angela.
   Stephen is taller than Khalid.
   Who is too small for the ride?

d. Jamie goes for lunch and buys a burger at £2, a coke at £1.20 and chips at £1.35 and pays with a £5. Jamie receives 5 coins in his change. What coins could they be? List all the different possible ways of receiving 5 coins in his change.

e. The Airdrop is a ride for 5 people.
   Ben is in the middle.
   Martina is on the outside of the car.
   Samir is between Ben and Matt.
   Claire is the other person in the car.

OUTSIDE ..... INSIDE

Copy this diagram to show the position of all the pupils in the Airdrop car.

f. Emilio and Katrina are playing a video game. Katrina has the high score but two of the digits are not clear. The high score says

Katrina gives Emilio two clues to help him find the high score.

Clue 1: 4 is the smallest digit.

Clue 2: All the digits add together to make 20.

Write all the possible missing pairs of digits.
Write all the possible scores that Katrina could have obtained.
What is the highest possible score that Katrina obtained?

g. On the way out of the park Katrina asks the attendant how many people entered the park that day. The attendant tells her that if the number was multiplied by 4, then divided by 1000 and then 9 was subtracted you got 1.

How many people entered the park?

Teachers Notes

AIMS To engage pupils in a range of problem solving strategies.

RESOURCES Pupils have two worksheets that state the problems and outline the tasks.
A warm up/lesson starter is included.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce the problems
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problems attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP To reinforce the word 'digit' and to use and organise lists.

STARTING Read through the problems with the pupils.
You may want to break the lesson down by reading and attempting one problem at a time.
Ask them to express the problems in their own words.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with each of the problems for a short time, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle similar problems again, which strategies would they use?
Could they improve on their strategies?

SOLUTIONS a. 152 pupils = 4 coaches at a cost of £400
or = 3 coaches (120 pupils) + 2 minibuses (32)
= £300 + £90 = £390. This is the cheapest.
or = 2 coaches (80) + 5 minibuses (80)
= £200 + £225 = £425
or = 1 coach (40) + 7 minibuses (112)
= £100 + £315 = £415
or = 10 minibuses = £450

b. AB, AC, AD
BA, BC, BD
etc
12 possible

c. Steven is taller than Khalid, who is taller than Angela.
Angela is too small for the ride.

d. Jamie receives 45p.
This could be 1 x 20p + 1 x 10p + 3 x 5p, or 4 x 10p + 5p,
or 2 x 20p + 2 x 2p + 1p
Each of these is a total of 5 coins.

e. Martina Claire Ben Samir Matt

f. The total of the missing digits is 20 – 9 = 11
2 + 9, 3 + 8, 4 + 7, 5 + 6, but 2 + 9 and 3 + 8 are eliminated since 4 is the smallest digit.
The possible numbers are:
4457 or 4754 or 4556 or 4655. The largest is 4754.

g. 2500 attended the park.

Warm-up activity/Lesson starter

Use the digits 5, 6, 7, 5 to make as many 4-digit numbers as possible with the number 7 first. For example, 7556.

What is the largest 4-digit number that can be made?

What is the smallest 4-digit number that can be made, with 7 still first?

What is the difference between these two numbers?

Section 6

Problem solving tasks and lesson plans

Level E

Contents

Teachers Notes

Resources

Each lesson has:

1. lesson starter materials
2. pupil worksheets
3. teacher notes
4. solutions.

Teacher notes highlight the main strategies that could be used in the problems.

The lesson starters are designed, whenever possible, to be related to the problem that will follow by highlighting possible strategies that could be used in the lesson and also to review vocabulary used in some problems.

Overheads are included for some problems to clarify diagrams or to allow games to be played by the teacher with the class.

The order in which the problems are tackled is not important but it is suggested that Small and Sweet could be tackled at the end as it requires a variety of strategies to be used and it could be used prior to pupils sitting their National Assessment.

Class structure

It is suggested that all of these problems should be tackled with the **pupils working in pairs or small groups.** Learning to reason requires interaction with someone else.

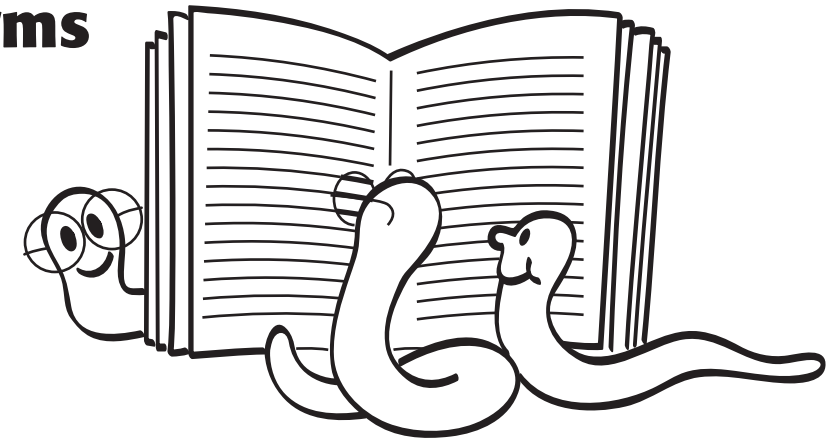
Pupils need to be challenged to think and talk about what they are doing when solving problems.

It should be stated explicitly to the class why they are in pairs or groups. It is so that they can discuss the problem and so that they can support each other. It is not just by accident that they have a partner.

Problems are best solved by pupils discussing with each other the possible strategies that can be used to tackle the problem and by supporting each other through the solving process.

Most pupils will learn more and be more confident attempting the problems using a paired or group approach rather than having to solve the problems on their own.

Bookworms



Three friends, Alfredo, Boris and Usma, enjoy reading and regularly go to the library to borrow books.

Alfredo reads fiction and returns to the library every 10 days.

Boris reads non-fiction and returns to the library every 15 days.

Usma uses the library to borrow DVDs and returns them every 40 days.

To their great surprise they are all in the library on the same day, Tuesday the 1st of June.

They decide to discuss when they will all be in the library on the same day again.

Alfredo thinks about it and says 'I don't think we will meet again this year.'

Boris guesses and says 'Never, that was only luck that we all met on the same day.'

Usma decides to use some maths and says 'See you on the 2nd of October.'

The problems

Are any of them correct? Work out if the three friends would meet again in the library.

If they do meet again work out the date and the day they would meet in the library

Extension

Use the formula to calculate the day of the week that you were born.

$$S = Y + D + \frac{(Y - 1)}{4}$$ Y = year D = day of year, Jan 1st = 1, Feb 1st = 32, etc

Step 1 Calculate S (ignore decimals).

Step 2 Divide S by 7 and note the remainder.

Step 3 If remainder is 0 it is a Friday.
If remainder is 1 it is a Saturday.
If remainder is 2 it is a Sunday.
If remainder is 3 it is a Monday, etc.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Reason logically
b. Make it simpler
c. Organise a list.

RESOURCES Pupils have two worksheets that state the problem and outline the tasks.
A warm up/lesson starter is included.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils on how they obtained their answers.
The lesson starter is to remind the pupils of their knowledge of months and days and so support the main problem solving activity.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.
c. use a diagram.

PLENARY SESSION The plenary sessions should be used to highlight the methods used. If they were to tackle a similar problem again, could they improve on their strategies?

SOLUTION The problem can be solved using lowest common multiples.
Multiples of 10 are: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120.
Multiples of 15 are: 15, 30, 45, 60, 75, 90, 105, 120.
Multiples of 40 are: 40, 80, 120.
The LCM of 10, 15, 40 is 120.
The friends do meet again.
They meet 120 days after the 1st of June.
30 days after 1st June is 1st of July.
30 days after the 1st of July is the 31st of July.
30 days after the 31st of July is the 30th of August.
30 days after 30th August is **29th of September.**
120 days is the same as 17 weeks and 1 day (120/7 = 17 remainder 1). From a Tuesday, 17 weeks later would be a Tuesday, plus one more day would make **Wednesday.**

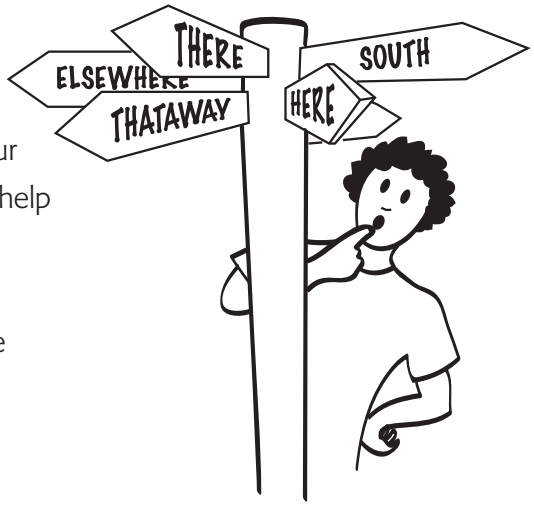
Warm-up activity/Lesson starter

1. How many days in total are in June and July?

2. Complete the following: 30 days has September, April, June and……..All the rest have……

3. Starting at 1st of August what would the date be in 7 weeks' time?

Confused?



The problems

The following problems should be discussed with your partner first to see if you can find a method that will help you both come to a solution.

They cannot be solved without some working and an explanation should be given as to how you found the solution.

Problem 1: Halloween

Susan, Leela, Richard and John are getting ready for a Halloween party. Each of them is dressing as a different character. They will go as a skeleton or a witch or a dragon or an alien.

- The dragon is a boy.
- The witch is not Leela.
- Sarah is not the skeleton.
- John is the alien.

Find out who is dressing up as what character.

Problem 2: The weddings

Joan, Sarah and Heather are engaged and are about to get married. The names of their future husbands are Gary, Michael and Paul.

From the information below can you find out who the couples are?

- Joan is not engaged to the engineer.
- Michael is a chef.
- The chef's future wife is not Heather.
- Gary is engaged to Heather.
- Paul is the engineer.

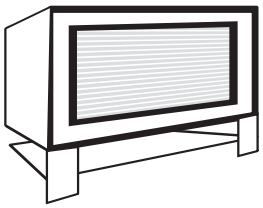
Problem 3: How old are you?

Paul, Mohammed, Narinda and Andrew are discussing their ages. They know that they are 28, 47, 50, 55 years old. From the clues below find out find how old they are.

- Mohammed is not the youngest.
- Narinda is not the oldest.
- Narinda's age is divisible by 5.
- Mohammed is younger than Narinda.
- Paul is younger than Mohammed.

Problem 4: TV addicts

Ahmed, Carmen, Permbir and Ryan want to watch TV. They check the newspaper and choose four different programmes. The programmes are King Kong, Eastenders, ET and The Goonies. From the clues below decide who wishes to watch what programme.



- Ryan wants to see The Goonies.
- Permbir does not want to see ET.
- Carmen does not like Eastenders.
- Ahmed does not like Eastenders or ET.

Extension

Can you create a similar problem for your partner to try? You should check first that your clues only give one possible answer.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Elimination
b. Be systematic
c. Draw a table.

RESOURCES Pupils have three worksheets that state the problem, outline the tasks and clarify the problem.
A warm up/lesson starter activity sheet is included.

TIME 1 to 2 periods

LESSON STRUCTURE a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP The purpose of the warm up is to allow the pupils to focus on thinking logically, using elimination and using a table to assist in the layout and the understanding of information given.

STARTING Read through the problems with the pupils.
Ask them to express the problem in their own words.
It is suggested that they discuss the problems in pairs before tackling the worksheet.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using.
Do they need further support to tackle the problems?

DOING Pupils should now continue with the problems, recording the steps as they progress through them.

PLENARY This time should be taken to allow the pupils to express how they tackled the problems, which parts were difficult and why they were difficult. Pupils should also be allowed to explain how they solved the problems.
If they were to tackle a similar problem again, which strategies would they use? Could they improve on their strategies?

Solutions Problem 1

Clue 1 The dragon is a boy: this eliminates Susan and Anne from the dragon column.

	Skeleton	Witch	Dragon	Alien
Susan			X	
Leela			X	
Richard				
John				

Clue 2 The witch is not Anne: this eliminates Anne from the witch column.

	Skeleton	Witch	Dragon	Alien
Susan			X	
Leela		X	X	
Richard				
John				

Clue 3 Susan is not the skeleton: this eliminates her from the skeleton column.

	Skeleton	Witch	Dragon	Alien
Susan	X		X	
Leela		X	X	
Richard				
John				

Clue 4 John is the alien: this eliminates all others from the alien column and John from the skeleton, dragon and witch columns. Richard must now be the dragon as other costumes are eliminated.

	Skeleton	Witch	Dragon	Alien
Susan	X	✓	X	X
Leela	✓	X	X	X
Richard	X	X	✓	X
John	X	X	X	✓

Susan must be the witch and Anne the skeleton.

Problem 2: The weddings

Again, using a table to lay out the information and to eliminate incorrect pairings is the best strategy. The additional information regarding occupation should be added to the table.

Joan is not engaged to the engineer.

Michael is a chef.

The chef is not married to Heather: this eliminates Heather and Michael as a pair.

Gary is married to Heather. This leaves only two pairs to be sorted

	Gary	Michael (Chef)	Paul
Joan	X		
Sarah	X		
Heather	✓	X	X

Paul is the engineer.

Since Joan is not engaged to the engineer, she is not engaged to Paul.

So she must be engaged to Michael.

	Gary	Michael (Chef)	Paul
Joan	X	✓	X
Sarah	X	X	✓
Heather	✓	X	X

Problem 3: How old are you?

Mohammed is not the youngest.
Narinda is not the oldest: this eliminates two boxes.
Narinda's age is divisible by 5, so her age must be 50
as we have already eliminated 55 as the oldest.

	28	47	50	55
Paul			X	
Mohammed	X		X	
Narinda	X	X	✓	X
Andrew			X	

Mohammed is younger than Narinda: this identifies his age as 47.
Paul is younger than Mohammed: this tells us that Paul must be 28.
This only leaves Andrew and the number 55.

	28	47	50	55
Paul	✓		X	
Mohammed		✓	X	
Narinda	X	X	✓	X
Andrew			X	✓

Problem 4: TV addicts

Ryan wants to see The Goonies.
Carmen wants to see ET.
Pembir wants to see Eastenders.
Ahmed wants to see King Kong.

Warm-up activity/Lesson starter

Use the clues to find the number.

1. The number is a four-digit number.
2. The numbers are 5, 6, 7, 8.
3. 5 is not the first digit.
4. 8 is not the last digit.
5. 7 is the second digit.
6. 6 is not the third or the fourth digits.

What is the number?

The table below will help you solve the problem.

	1st	2nd	3rd	4th
5				
6				
7				
8				

Crazy Digits

The problems

Tom has forgotten his PIN number for his bank card.

However, he remembers various clues to his number.

Task 1

Use the clues below to find the missing number:

- The number has four digits.
- The product of the extreme digits is 36.
- The product of the middle digits is 24.

Write down all the possible four-digit numbers that could be Tom's PIN number.

Task 2

Tom is still confused, but he then remembers some more clues:

- The unit digit and the thousand digits are not the same.
- The sum of the digits is 23.

Write down all the possible four-digit numbers that could be Tom's PIN number now.

Task 3

He also remembers that the number is even and that the unit digit is smaller than the tens digit.

He is now confident enough to use his bank card.

Write down the correct PIN code for Tom's bank card.

Extension

Go back to the end of task 2. There were 4 possible different PIN numbers.

Tom knows that he could guess the number. What is the probability of correctly guessing the correct PIN number if there are 4 possible solutions? If you guessed incorrectly and you decide to guess again, what is your chance of guessing correctly this time? If you are wrong again, what would your chance be of guessing correctly this time? What happens to your card if you guess incorrectly a third time?

At the start of the problem, when Tom could not remember his PIN number and he could not remember any clues, what would be the probability of guessing his four-digit PIN number correctly?

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Organise a list
b. Elimination.

RESOURCES Pupils have a worksheet that states the problem and outlines the tasks.
A warm up is included.

TIME 1 period

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils on how they obtained their answers.
The warm up is to remind the pupils of the language used in the problem.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
Ensure that pupils understand language used in the problem, e.g. unit, extreme, middle digits, product, sum of.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.** This is an important part of the learning process and it perhaps can be difficult for most teachers not to intervene too soon. However, pupils require thinking time to discuss their own ideas and formulate possible solutions.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using.

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be encouraged to explain their solutions to the rest of the class, paying particular reference to the strategies used.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION Task 1

The one-digit factors of 36 are: 9 and 4, 4 and 9, 6 and 6
The one-digit factors of 24 are: 8 and 3, 3 and 8, 4 and 6, 6 and 4
The possible numbers are:
9834, 9384, 9464, 9644
4839, 4389, 4469, 4649
6836, 6386, 6466, 6646

Task 2

Eliminate all the possible numbers starting with 6 and ending in 6.

9834, 9384, 9464, 9644

4839, 4389, 4469, 4649

The sum of the digits is 23: this gives the following possible numbers:

9464, 9644, 4469, 4649

Task 3

Even number, so 9464 and 9644 are left.

The answer is 9464, since in 9644 the units and tens digit are equal.

EXTENSION The extension is to allow the pupils to explore the real-life situation further.

Warm-up activity/Lesson starter

Use number 8932 to answer the following questions:

a. Find the sum of the units and the tens digit.

b. The digits at the extreme of this number are 8 and 2.
What is the product of the digits at the extreme of the number?

c. Subtract the thousand digit from the tens digit.

d. Using the digits 8, 9, 3, 2 what is the largest four-digit number that can be made?

Crossing the River

Anne, Bob, Cara, and Dawalli are on a camping trip. As part of the trip they have to cross a river on a raft.
The raft can only take a maximum of 100kg before it would start to sink.
The weight of Anne is 74kg. Bob's is 49kg. Cara's is 58kg. And Dawalli's is 39kg.

The problems

What is the least number of crossings that need to be made by the raft to get the entire group to the other side?

Pupil Worksheet

Task 1

Discuss with your partners what strategies you could use to attempt to solve the problem.

Task 2

Write down all the possible combinations of people who can cross the river in the raft at the same time.

Task 3

Attempt to solve the problem by acting out the problem, or by making models if possible.
Remember to record your moves as you go through the problem.

Task 4

Record your solution of the problem.
You could use diagrams to help your explanation of how you solved the problem.

Extension

Eddie joins the group and he weighs 42kg.
What is the least number of crossings that the raft will make to enable the whole group to cross the river?

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:

a. Act it out
b. Draw a diagram
c. Think logically
d. Make a model
e. Guess and check.

RESOURCES Pupils have two worksheets that state the problem and outline the tasks.
A lesson starter is included.

TIME 1 to 2 periods

LESSON STRUCTURE

a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils on how they obtained their answers.
The starter is to encourage the pupils to think logically.

STARTING **The problem solving activity would be enhanced if the pupils could be arranged in groups of four to allow them to act out the problem.**
Read through the problem with the pupils.
Ask them to express the problem in their own words.
Ensure that pupils understand the process of crossing the river and importantly that they need someone to take the raft back each time.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using.
This is now an opportunity to ask if they are finding the problem difficult, and if so to offer more support.

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:

a. work systematically

b. record the results in an organised manner as they work through the problem.

The task would ideally be attempted in a group of four, which could allow the pupils to act out the situation if possible.

Pupils will have to make more than one attempt at this problem before they decide what is the lowest number of crossings.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.

Pupils should be encouraged to explain their solutions to the rest of the class, paying particular reference to the strategies used.

If they were to tackle a similar problem again, which strategies would they use? Could they improve on their strategies?

SOLUTION Task 2

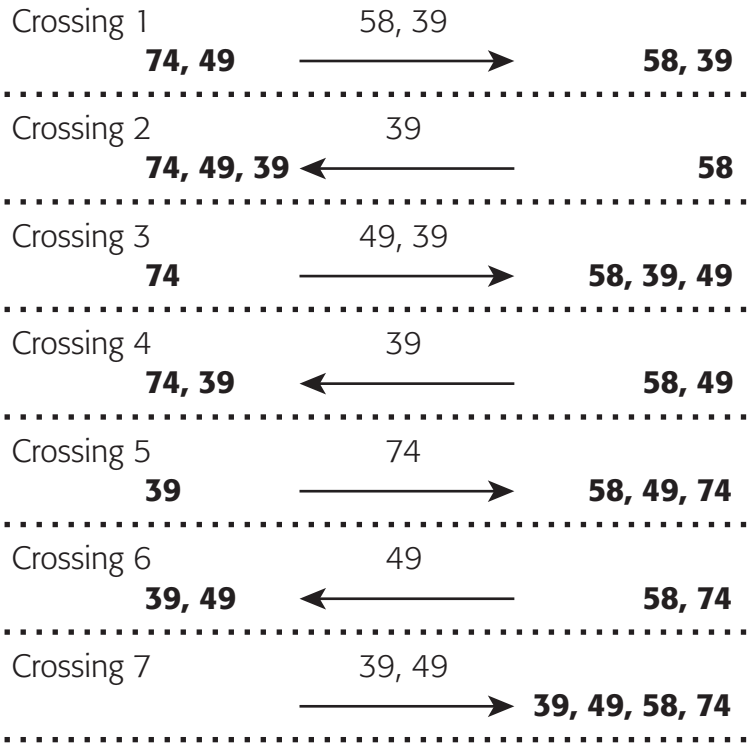
Bob can cross with Dawalli, a total of 88kg.

Cara can cross with Dawalli, a total of 97kg.

Task 4

There are a number of different ways to solve the problem.

This is one possible solution. The weights, rather than the names, have been used for ease of explanation.

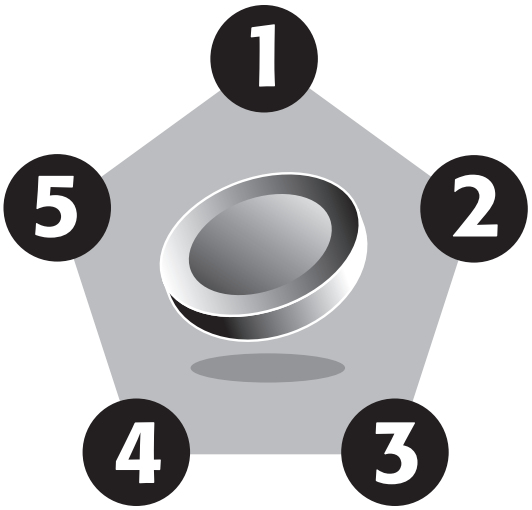


Warm-up activity/Lesson starter

Using the numbers 3 and 4 only and the symbols x, –, and =, make the numbers from 1 up to 10.
The digits 3 and 4 and the symbols x and – can be used as often as needed but no other digits can be used.

Example
4 – 3 = 1
4 x 3 – 4 = 8
3 x 3 – 4 = 5, etc.

Fifty-niner



Mr. Falconer, the evil maths teacher, challenges his class to a game of Fifty-niner. He says that if he wins all the games then the class will get double homework.
To the horror of the class Mr. Falconer wins all the games and the homework is doubled.

The rules of Fifty-niner

It is a game for two people.
Each pupil takes it in turns to move a counter to any of the numbers 1, 2, 3, 4, or 5 and keeps a running total.
For example:
I go first and move to 3. My opponent moves to 1. The total is now 4. I go again and move the counter to 5.
The total is now 9. My opponent moves to 3. The total is now 12.
The winner of the game is the first person to reach 59.

The problems

Explain, with examples, what strategy Mr. Falconer would use that could allow him to win every game.

Task 1

With a partner play the game of Fifty-niner. Take turns in going first.
The game sheet is on the next page.

Task 2

With your partner use the same game sheet, but this time the winning total is to be 13.
The numbers to be added on are still 1, 2, 3, 4 or 5.
Take turns going first.
Find a strategy that would allow you to win each time.
Explain this strategy with examples.

Task 3

With your partner play a similar game, but this time the winning total is to be 20. The numbers to be added on are still 1, 2, 3, 4 or 5.
Take turns going first.
Find a strategy that would allow you to win each time.
Explain this strategy with examples.

Task 4

Can you now come up with a strategy that will allow you to win at Fifty-niner? Play the Fifty-niner game again if you wish. Explain your strategy with examples.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:
a. Work backwards
b. Act it out
c. Make it simpler.

RESOURCES Pupils have three worksheets that state the problem, outline the tasks and allow the pupils to play the game.
An overhead transparency is provided to assist the introduction of the problem and to allow the teacher to play the game with the pupils.
A warm up is included.

TIME 1 to 2 periods

LESSON STRUCTURE
a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils on how they obtained their answers.
The starter is to encourage the pupils to use the working backwards strategy.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
Playing the game with the pupils using an overhead would beneficial before the pupils start the worksheets.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using.
Are the pupils able to play the games without further support?
Can they start to tackle the problem?
Is further discussion required to support the pupils to come up with a strategy to explain their method for winning the game?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Playing the game on the overhead again would be an ideal way of allowing pupils to demonstrate their strategies.
If they were to tackle a similar problem again, could they improve on their strategies?

SOLUTION **Task 2**

By working backwards pupils will realise that if your opponent gets to a total of between 8 and 12, then you will be able to win the game in your next turn by choosing 5 or 4 or 3 or 2 or 1 for your number.

This can be done if you get a total of 7 and going back again if you start at 1.

So to win: Go first. Move to 1. Next move go to 7. Next move you will win.

Task 3

A similar approach to a total of 20 will lead to:

Go first.

Start at 2, go to 8, then 14, then 20 on your next move.

Task 4

For 59:

Start at 5 and then 11, 17, 23, 29, 35, 41, 47, 53 and then 59.

Warm-up activity/Lesson starter

Solve the puzzles below:

a. I think of a number and add 6. I now have 13.

b. What was the number I started with?

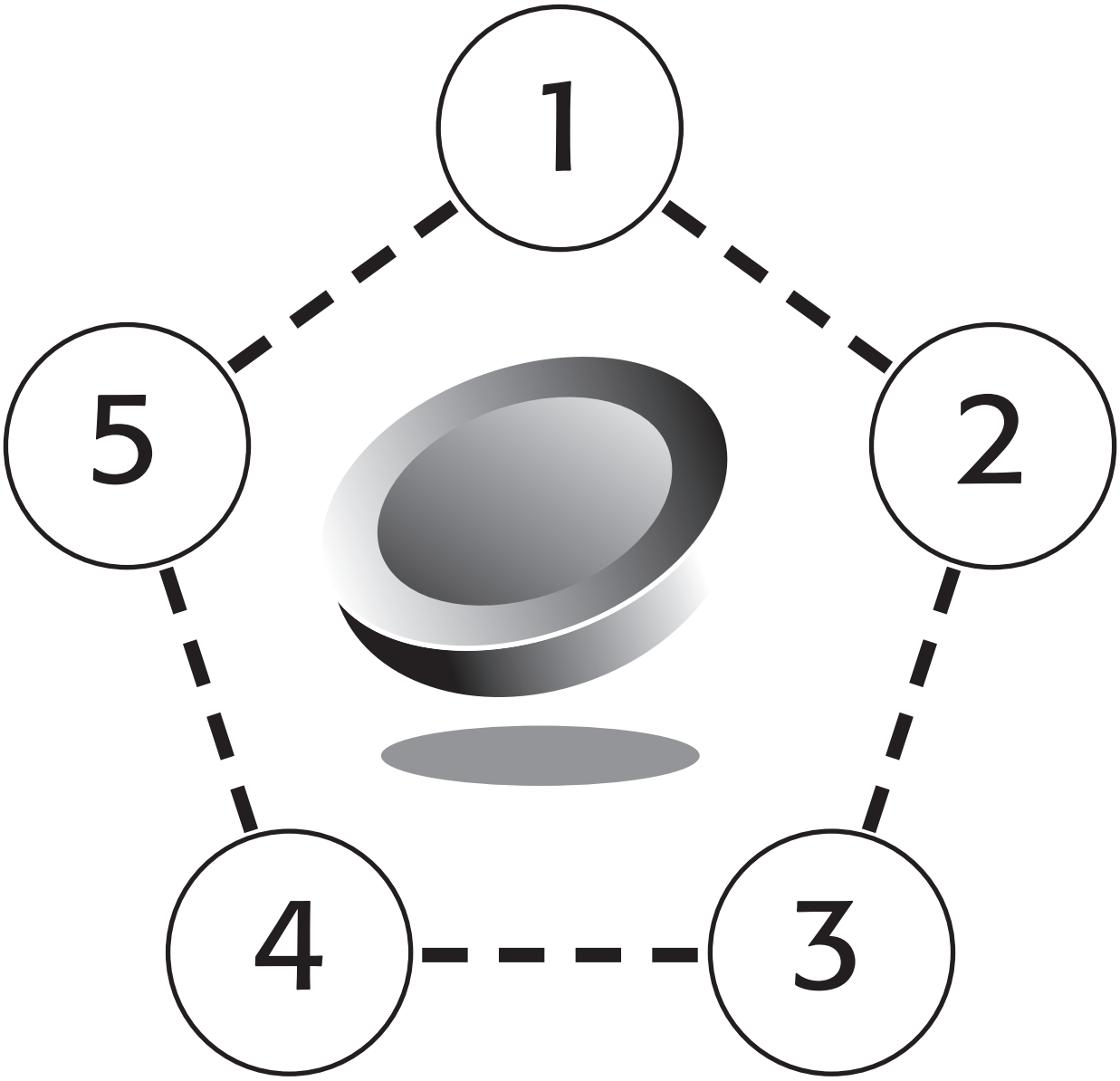
c. I think of a number and subtract 11.4. I now have 10. What was the number I started with?

d. I think of a number, multiply it by 10, and then add 40. I now have 190. What was the number I started with?

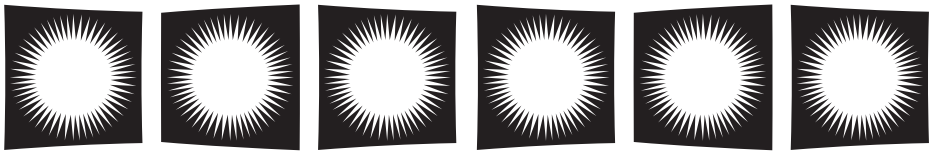
e. I think of a number, divide it by 100, and then subtract 4. I now have 6. What was the number I started with?

f. I think of a number, add 4, then multiply it 5. I now have 65. What was the number I started with?

Overhead Transparency



Lights



A local nightclub has installed a new lighting system.
The lighting system consists of eight different lights, with each light a different colour.
The club's DJ controls the lights with the music system. He can have each light either on or off.
The DJ, who didn't do maths at school, complains that he only has eight different combinations of lights and that this will not be enough to create a good atmosphere in the club.
The DJ has a daughter in second year in school who tells her dad that he is daft because with eight lights which can be either on or off, there are more than 200 different combinations.

The problems

Find out how many different ways eight lights can be arranged, either on or off.

Pupil Worksheet

Task 1

With a partner discuss what strategies could be used to help tackle this problem.
Write down the different strategies you will use to attempt the problem.

Task 2

If there are only 2 lights the lights could be:

OFF	ON	ON	ON
OFF	OFF	ON	OFF

There are only four different combinations of either OFF or ON

If there are only 3 lights write down all the different combinations of on and off.

Task 3

Discuss the problem with your partner again.
Now try to solve the problem, i.e. in how many different combinations of the lights being on or off can eight lights be arranged?

Extension

Find a formula that could be used to calculate how many different ways any number of lights can be arranged either on or off.
Use the formula to calculate how many ways 20 lights can be arranged either on or off.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:

a. Try a simpler case
b. Be systematic
c. Draw a table
d. Look for a pattern
e. Use a formula.

RESOURCES Pupils have two worksheets that state the problem and outline the tasks.
A warm up is included.

TIME 1 to 2 periods

LESSON STRUCTURE

a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils on how they obtained their answers.

STARTING Read through the problem with the pupils.
An initial explanation is essential to ensure that the pupils understand the problem.
Ask them to express the problem in their own words.
It is suggested that they discuss the problems in pairs before tackling the worksheet.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:

a. work systematically

b. record the results in an organised manner as they work through the problem.

A discussion on the best way to record the results and the use of tables to spot patterns would be valuable to many pupils.

The extension should be accessible to most pupils no matter what approach they used.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.

Pupils should be encouraged to explain their solutions to the rest of the class, paying particular reference to the strategies they have used.

The use of a formula could be highlighted through the extension.

If they were to tackle a similar problem again, could they improve on their strategies?

SOLUTION **Task 3**

There are eight possible combinations:

on on on

on on off

off on on

on off on

on off off

off on off

off off on

off off off

SOLUTION **Task 4**

One way to tackle the problem is to draw a table and spot the pattern:

Number of lights	1	2	3	4
Number of combinations	2	4	8	16

To check the pattern, an organised list of 4 lights should be drawn.

on on on on on on on off on on off on on off on on off on on on

on on off off on off on off on off off on off on on off off on off on

off off on on on off off off off on off off off off on off off off off on

off off off off

16 different combinations

Number of lights	5	6	7	8
Number of combinations	32	64	128	256

The solution is 256 different combinations.

EXTENSION nth Term $= 2^n$

The pupils may not have covered powers by level E.

They should be encouraged to express the formula in any appropriate language.

For example, the multiples of two, or the two times table, would be a good way of expressing the formula.

20 lights $= 2^{20}$

Warm-up activity/Lesson starter

In the sequences below find the next two numbers and state the rule.

Example:

301, 402, 503, 604, 705. The rule is add 101.

404, 302, 200,

20, 60, 180,

4, 2, 1,

1, 4, 9,

3, 9, 27,

School Yard

Class 1.3 complained to the head teacher that they have nowhere to play football in the schoolyard. Class 1.4 complained that they want to play basketball but have no space and 2.4 complained they have nowhere to sit in the school yard.

In the end the head teacher agrees that every class in S1, S2, S3 and S4 will have their own separate area in the playground.

There are 29 classes in S1 to S4.

A pupil in S1 tells the head teacher that the school janitor only has to paint 7 lines on the circular playground to make 29 separate spaces.

The head teacher disagrees, pointing out that many more lines would be required to give each class its own separate area.

The problems

Is the pupil correct? How many lines are required to be drawn in a circle so that it will be divided into 29 areas?

Pupil Worksheet 1

Task 1

With a partner discuss what strategies you could use that would help you solve the problem.

Task 2

It is known that a circle with three lines crossing it can be divided into a maximum of 7 areas.

Use the circle sheet attached to draw three lines on a circle so that it is divided into 7 separate areas.

Task 3

It is also known that a circle with four lines crossing it can be divided into 11 areas.

Use the circle sheet attached to draw four lines that will divide the circle into 11 separate areas.

Task 4

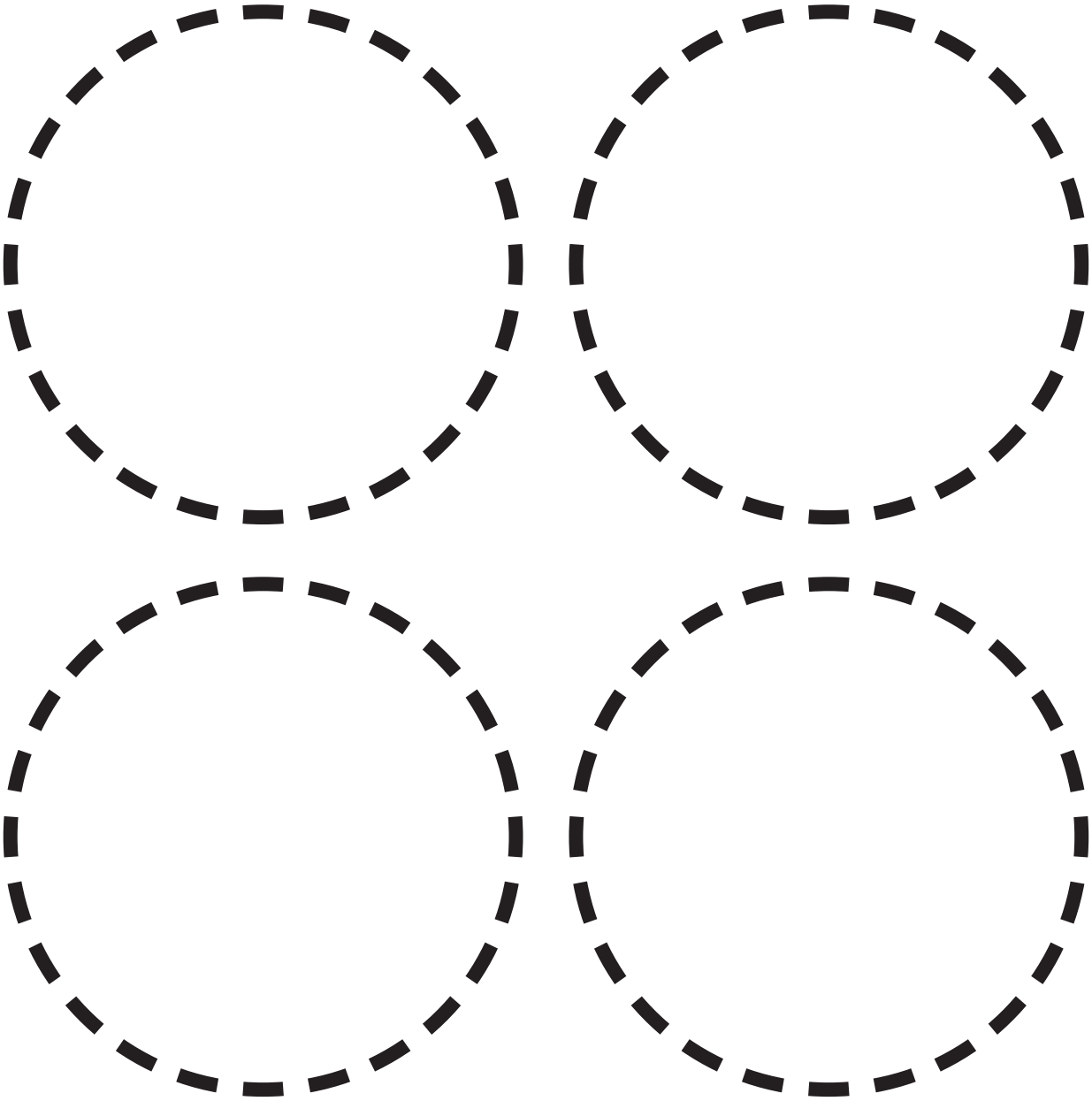
Now discuss again with your partner what strategies you could use that would help you solve the problem.

Would 7 lines on a circle divide it into 29 areas?

Extension

Find a formula connecting the number of lines and the number of separate areas.

Pupil Worksheet 2



Teachers Notes

AIMS To engage pupils in the following problem solving strategies:

a. Try a simpler case
b. Draw a diagram
c. Draw a table
d. Look for a pattern
e. Trial and error.

RESOURCES Pupils have three worksheets that state the problem and outline the tasks.
An overhead transparency is provided to assist the introduction of the problem and to clarify dividing the circle into separate areas.
A warm up is included.

TIME 1 to 2 periods

LESSON STRUCTURE

a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils on how they obtained their answers.
The starter is to encourage the pupils to spot a pattern.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
Ensure that pupils understand the process of dividing up a circle into separate areas using lines.
Pupils will need to persevere to find the solution to tasks 2 and 3.
They are using trial and error when attempting to divide the circle into the maximum areas.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.** This is an important part of the learning process and it perhaps can be difficult for most teachers not to intervene too soon. However, pupils require thinking time to discuss their own ideas and formulate possible solutions.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using.

It would be difficult for pupils to solve the problem by using a diagram and counting the areas. This should form part of the teacher/pupil discussion and highlight the need to use more than one method to solve a problem.

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:

a. work systematically

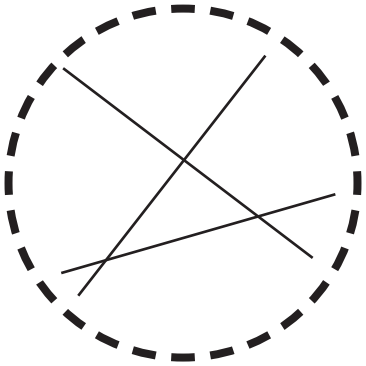
b. record the results in an organised manner as they work through the problem

c. use a table to spot the pattern.

It is unlikely that pupils could accurately draw all the separate areas.

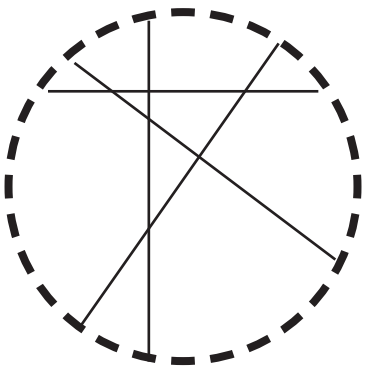
PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be encouraged to explain their solutions to the rest of the class, paying particular reference to the strategies used.
If they were to tackle a similar problem again, which strategies would they use? Could they improve on their strategies?

SOLUTION **Task 2**



Many solutions are possible as long as the 3 lines do not intersect at a common point

Task 3



Task 4

Drawing a table should allow the pupils to continue the pattern and solve the problem.

No. of Lines	1	2	3	4	5	6	7
No. of Areas	2	4	7	11	16	22	29

SOLUTION The pupil is correct. A circle will give 29 areas with 7 lines.

Warm-up activity/Lesson starter

Complete the sequences

a. 2, 4, 6,……, 14, 16

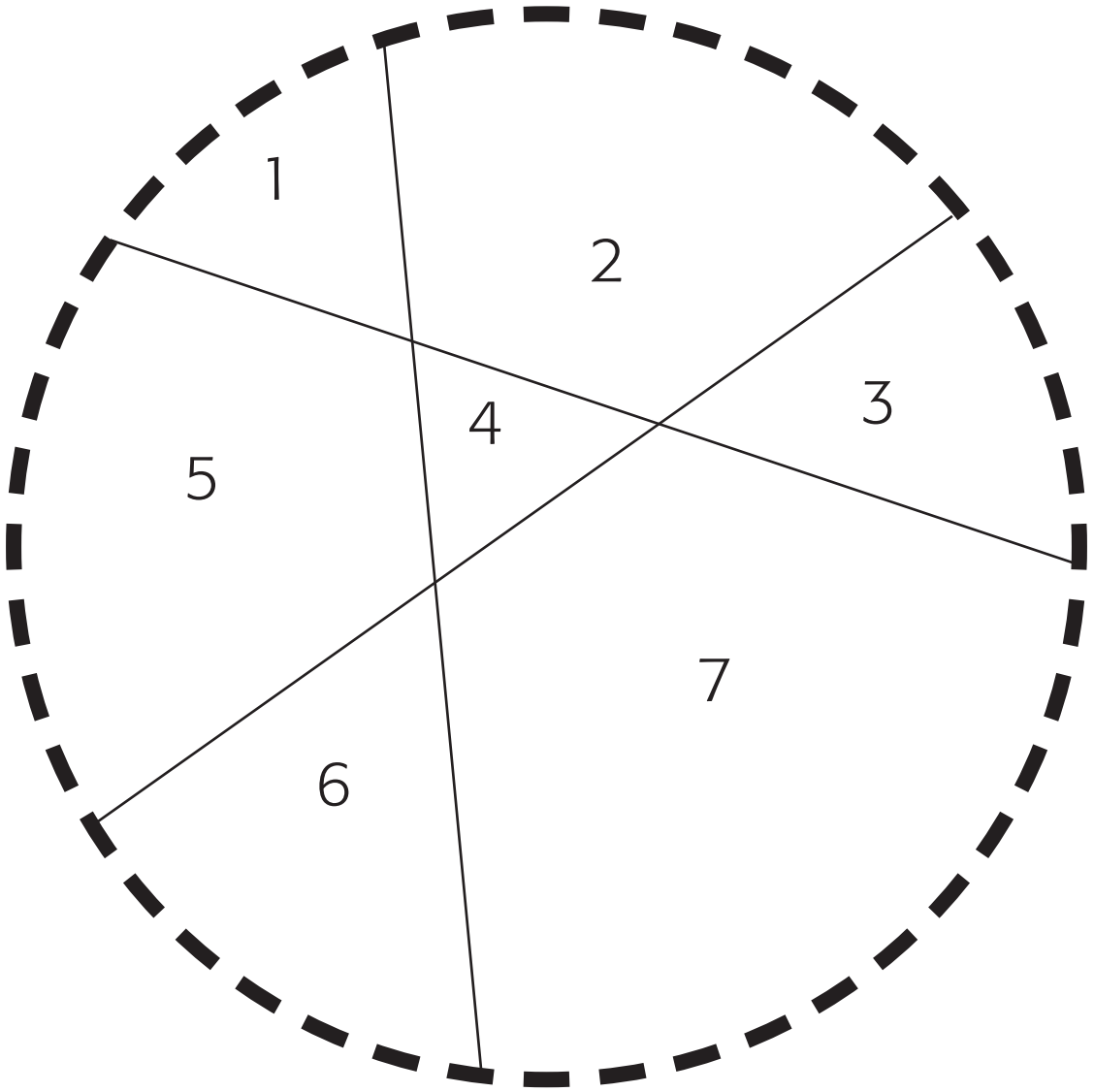
b. –1, –3, –5,……–13, –15

c. 2.3, 3.4, 4.5,……, 8.9

d. 1, 2, 4, 7, 11,……., 36, 45

e. 3, 5, 8, 12,……, 30

Overhead Transparency



Spy-catcher

The problems

A government agent is gathering information about an enemy spy so that the spy can be caught.

Your mission is to use the clues given to find the name of the spy, the country, the town and the address of the spy's hideout.

Clue 1

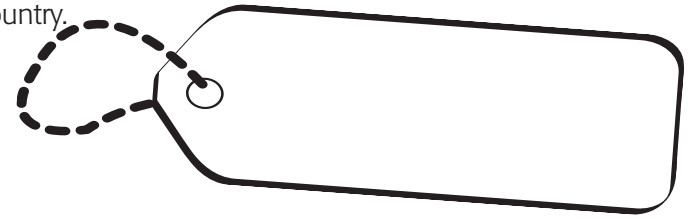
The spy has been spotted moving from one country to another.

The spy has recently been in the following countries:

Use the clues below to find out which country, from the above list, the spy is in now.

a. The country's name does not start with a vowel.
b. The country's name starts with a letter that has either rotational symmetry or line symmetry.
c. The number of letters in the country's name is not a prime number.
d. The number of letters in the country's name is divisible by 4.

Write the down the name of the country.



Clue 2

The spy is known to have a number of safe houses throughout the country.

Before the safe house can be found you have to be sure that you know the number of safe houses. Use these clues to find the number of safe houses:

a. The number is between 20 and 60.
b. The number is odd.
c. The number is not a prime number.
d. The sum of the number's digits is less than 5.
e. How many safe houses does the spy have?

Clue 3

The town where the spy is hiding can be found from the following information.

a. The town has 6 letters.
b. The 1st letter and the 4th letter are the same and are the 4th letter in the alphabet.
c. The 5th and 6th letters are the same vowel (AEIOU). The letter has horizontal line symmetry only and does not have rotational symmetry.
d. The third letter of the town's name is the 14th letter of the alphabet.
e. The town is in Scotland.

Clue 4

The name of the street the spy is hiding is known.

It is Spey Road.

Your mission is to use the clues below to find the number of the house in Spey Road where the spy is hiding.

a. The house number is even and has 2 digits.
b. The house number is a square number.
c. The number is a multiple of 9.

What number on Spey Road is the spy hiding at?

Clue 5

The first name of the spy is either Henrik, James, Sam, John or Sylvester.

The spy's second name is either Bond, Stallone, Larsons, or Greig.

Clues:

a. The second name has less than 7 letters.
b. The second name has 2 letters more than the first name.

Write down the name of the spy.

CONGRATULATIONS! YOU HAVE CAUGHT THE SPY.

Report

Write a report to your superior detailing everything you now know about the spy.

The report could include where they were caught, what they were spying on and how you managed to catch the spy.

Make sure you give yourself a cool spy name.

Extension

Can you make your own puzzle that will be solved by your partner?

The puzzle is to find out the age of the spy.

You have to give enough clues so that when they are solved they will only give one number.

Try to give as few clues as possible so that it is not too easy to solve.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:

a. Elimination
b. Be systematic
c. Organised list.

Pupils are asked to write a report at the end of this problem.
This is an opportunity to link maths with the outside world and to the world of work. It is also an opportunity to liaise with the English Department on a piece of imaginative writing or on report writing.

RESOURCES Pupils have four worksheets that state the problem and outline the tasks.
A warm up activity is included.

TIME 1 period

LESSON STRUCTURE

a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP The main focus of the warm up is to support the vocabulary used in the problem.
Pupils **must** know the meaning of:
line, rotational, horizontal and vertical symmetry
odd numbers, prime numbers, square numbers, multiples, and digit.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
It is suggested that they discuss the problems in pairs before tackling the worksheet.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using.
The main strategy is to write down an organised list of all the possible solutions and then eliminate all the incorrect possibilities to find the solution.
It may be necessary to highlight these strategies if pupils are encountering difficulties.

DOING Pupils should now go on to solve the problem. They should be encouraged to:

a. work systematically

b. record the results in an organised manner as they work through the problem.

A discussion on the best way to record the results and the use of lists would be valuable to many pupils.

The extension is to encourage further discussion of different types of numbers between pupils.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.

Pupils should be encouraged to explain their solutions to the rest of the class, paying particular reference to the strategies they have used.

It is also a further opportunity to re-enforce the vocabulary used throughout the problem.

SOLUTION **Clue 1**

a. Uganda and Afghanistan are eliminated as they are vowels.

b. Kenya and Poland are eliminated as they have no symmetry.

c. Chile has 5 letters and is eliminated as 5 is a prime number.

The solution is Scotland.

Clue 2

a. 20 to 60

b. 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59

c. 21, 25, 27, 33, 35, 39, 45, 49, 51, 55, 57

d. The sum of the digits is less than 5.

The solution is 21.

Clue 3

a. - - - - - -

b. D- - - D- -

c. D - - D E E

d. D- N D E E

e. DUNDEE

Clue 4

a. 10,12, 14,16……98

b. 16 36 64

c. 36

The solution is 36.

Clue 5

a. Bond or Greig have less then 7 letters.

b. Bond has 4 letters. There are no first names with 2 letters so Bond is eliminated. Greig has 5 letters so the first name has 3 letters. So the name is Sam.

The solution is SAM GREIG.

Warm-up activity/Lesson starter

These are the vowels.

Copy them down and draw in:

a. the horizontal lines of symmetry

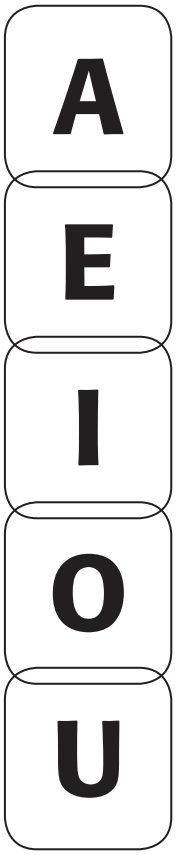
b. the vertical lines of symmetry.

2. Write down the letters which have rotational symmetry.

3. Write down the numbers from 1 to 17.

a. From this list write down all the square numbers.

b. For which square number, from the list, do the digits add to 7?



Stepladders

Ismael is painting the walls and ceilings in a house and is using a ladder to help.
The ladder has ten rungs.
Ismael can step up either one or two rungs at a time.
He cannot step up more than two rungs at a time because he knows this would be dangerous.
To stand on the second rung he could

a. go up taking one at a time, i.e. 1, 1 or
b. go up taking two at a time, i.e. 2.

So there are two different ways for Ismael to go up to the second rung.

The problems

Ismael wants to stand on the tenth rung to paint the ceiling.
How many different ways can he climb to the tenth rung?

Pupil Worksheet 1

Task 1

With a partner discuss what strategies you could use that would help you solve the problem.

Task 2

If the ladder had only three rungs how many different ways could Ismael go up the ladder?

Task 3

If the ladder had only four rungs how many different ways could Ismael go up the ladder?

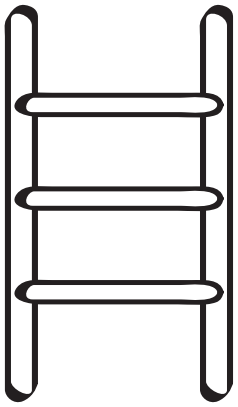
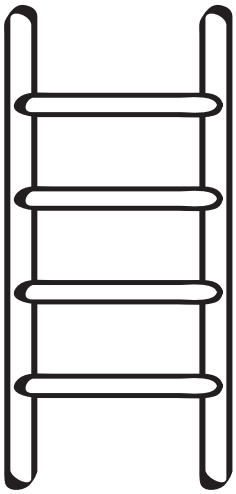
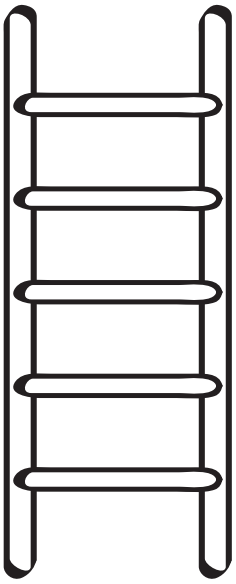
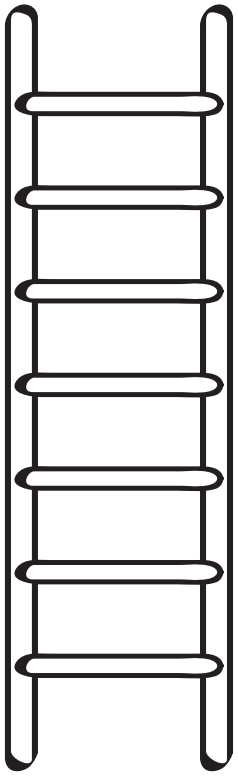
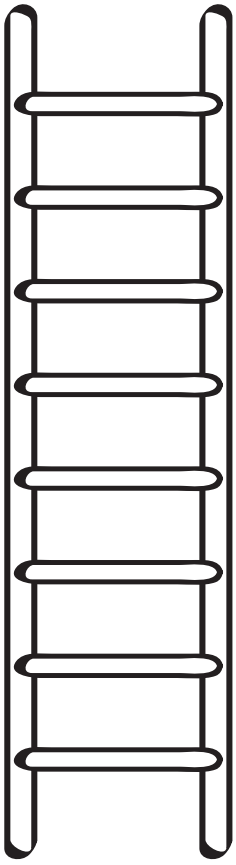
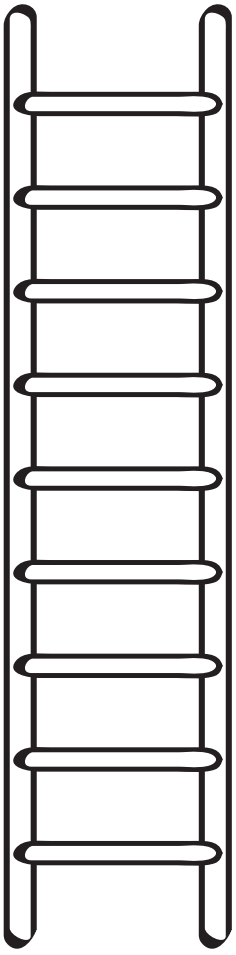
Task 4

Now attempt to solve the problem of the ladder with ten rungs.
You can use the worksheet attached to help you if you wish.

Extension

What if there were 100 steps and you could jump 1 or 2 or 3 steps at a time? How many different ways could you climb the ladders?

Pupil Worksheet 2

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:

a. Try a simpler case
b. Be systematic
c. Draw a table
d. Look for a pattern
e. Act it out.

RESOURCES Pupils have three worksheets that state the problem and outline the tasks.
An overhead transparency is provided to assist the introduction of the problem and to clarify the stepping up the ladder procedure.
A lesson starter is included.

TIME 1 to 2 periods

LESSON STRUCTURE

f. Warm up (5 mins)
g. Introduce problem
h. Pupils discuss in pairs/groups and start (5–10 mins)
i. Teacher stops class and checks progress, intervenes if necessary (5 mins)
j. Problem attempted by pupils, teacher supports individual pupils (15 mins)
k. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils on how they obtained their answers.
Did any pupils make the questions simpler by changing the order of the additions?
Pupils should be reminded that making problems simpler is one method that can be used to solve difficult problems.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
Ensure that pupils understand the process of stepping up the ladder in 1 or 2 steps.
There is an opportunity to **act out** the problem by using the ladder worksheet as an overhead and moving counters up and down the ladders.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using.

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:

a. work systematically

b. record the results in an organised manner as they work through the problem.

A discussion on the best way to record the results and the use of tables to spot patterns would be valuable to many pupils.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.

Pupils should be encouraged to explain their solutions to the rest of the class, paying particular reference to the strategies used.

The problem was difficult until they tried a **simpler case.**

By listing the steps **systematically** it was easier not to miss any.

When the number of different ways to climb the ladder became very large what strategy could be used to help?

If they were to tackle a similar problem again, could they improve on their strategies?

SOLUTION Two steps

1,1 2 2 ways

Three steps

1,1,1 2,1 1,2 3 ways

Four steps

1,1,1,1 1,1,2 1,2,1 2,1,1 2,2 5 ways

Five steps

1,1,1,1,1 1,1,1,2 1,1,2,1 1,2,1,1

2,1,1,1 2,2,1 2,1,2 1,2,2 8 ways

Six steps

1,1,1,1,1,1 1,1,1,1,2 1,1,1,2,1 1,1,2,1,1 1,2,1,1,1

2,1,1,1,1 2, 2,1,1 2,1,2,1 2,1,1,2 1, 1, 2, 2 1, 2, 1, 2

1, 2, 2, 1 2, 2, 2 13 ways

Pupils could continue to list in an organised manner but a more efficient method would be to make a table and spot a pattern.

No. of steps	No. of ways of climbing
2	2
3	3
4	5
5	8
6	13
7	21
8	34
9	55
10	89

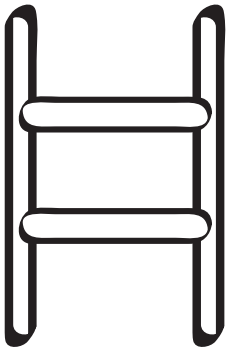
The solution to the problem is that there are 89 ways to go up 10 steps.

Warm-up activity/Lesson starter

4.6	6.5	9.7	5.4	3.3
6.2	1.9	7.8	3.8	6.1
1.8	7.1	2.2	6.9	2.6
4.4	3.5	1.3	9.6	2.7

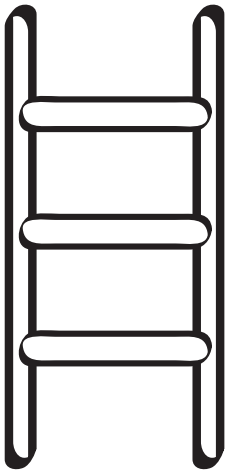
1. Add the numbers in the first column.
2. Add the numbers in the middle column.
3. Add the numbers in the first row.

Overhead Transparency



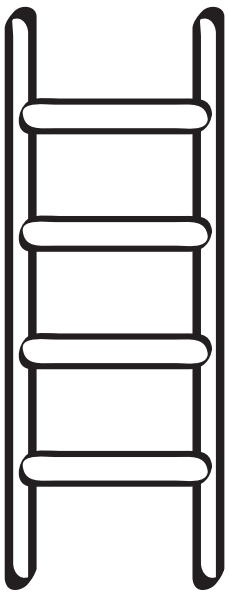
No. of steps 2

No. of ways of climbing



No. of steps 3

No. of ways of climbing



No. of steps 4

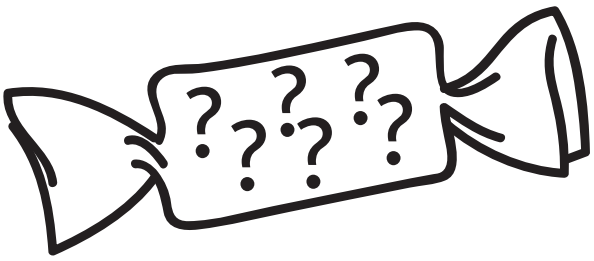
No. of ways of climbing

Small and Sweet

The problems

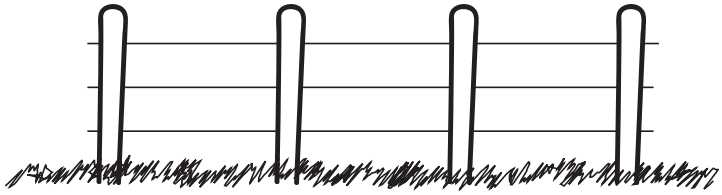
The problems below can be tackled using the problem solving techniques you have learned and practised.

For each problem, firstly discuss with your partner which strategies will help you tackle the problem and then lay out your working systematically, and record the strategy you used.



Problem 1

A builder is constructing a new fence like the one below.



The fence posts are the vertical lines.

The rails are the horizontal lines.

For 2 fence posts the builder needs 3 rails.

How many rails would he need for 200 fence posts?

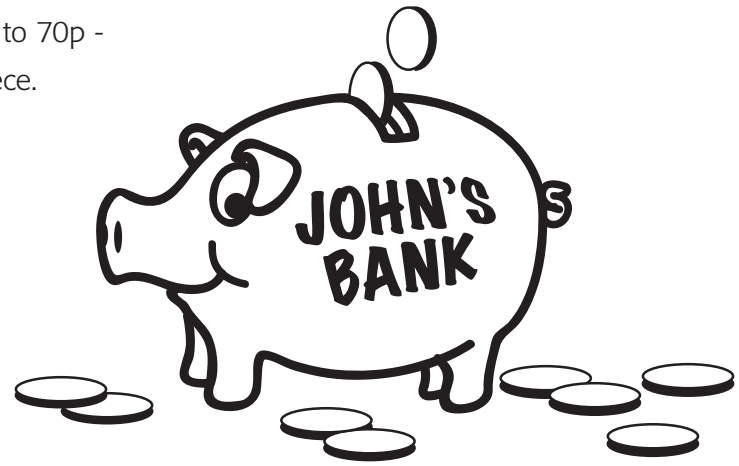
Write down a formula connecting rails and posts.

Problem 2

a. John has the same number of 10p coins and 50p coins. The total value of all the coins is £18. How many 10p coins does John have?

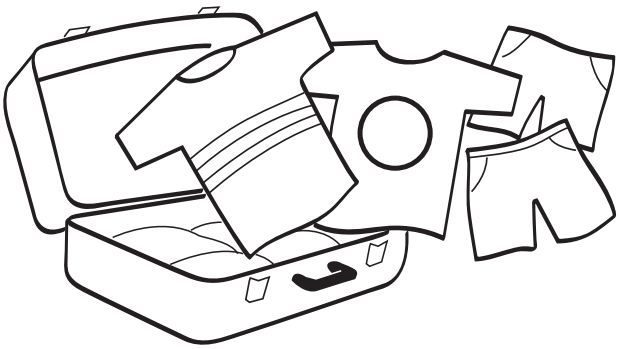
b. Suhail has the same number of 5p coins, 10p coins and 20p coins. The total value of the coins is £3.50. How many 5p coins does Suhail have?

c. I have 2 coins which add up to 70p - one of them is not a 50p piece. What coins do I have?



Problem 3

Abdul and David are buying new clothes for their holiday in Spain.
Abdul buys 2 T-shirts and a pair of shorts, while David buys 3 pairs of shorts and a T-shirt.



Surprisingly they discover they have spent the same amount of money. If a T-shirt costs £9 what would a pair of shorts cost?

Problem 4

All workers in an office have a three-digit security code that allows them access to their computer.
Neil has forgotten his.
He remembers that the numbers are consecutive.

a. Write down all the possible numbers that the security code could be.
b. He now remembers that the middle digit is a prime number. Write down all possible numbers the code could be now.
c. Finally he knows the security code after he remembers that the final digit is a squared number.

What is Neil's security code?

Problem 5

Caitlin withdraws £30 from the cash machine. The cash machine dispenses £5, £10 and £20 notes.
List all the different ways the autoteller could issue £30.
For example, four £5 notes plus a £10 note.

Teachers Notes

AIMS To allow the pupils to engage in problems similar to that found in the National Tests.

RESOURCES Pupils have two worksheets that state the problem and outline the tasks.
A warm up is included.

TIME 1 period

LESSON STRUCTURE
c. Warm up (5 mins)
d. Introduce problem
e. Pupils discuss in pairs/groups and start (5–10 mins)
f. Teacher stops class and checks progress, intervenes if necessary (5 mins)
g. Problem attempted by pupils, teacher supports individual pupils (15 mins)
h. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils to ensure that they understand the language used.
For example, consecutive, prime, digit, and square numbers are all used in the problems.

STARTING Read through the problem with the pupils.
A review of some of the language would be helpful, for example consecutive numbers, square numbers, prime numbers, digit, etc.
Ask them to express the problem in their own words.
It is suggested that they discuss each problems in pairs before tackling the problem.

DOING Pupils should attempt to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be encouraged to explain their solutions to the rest of the class, paying particular reference to the strategies they have used.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTIONS **Problem 1**

Pupils may use a number of strategies to solve the problem.
This could include make a table, spot a pattern, find a formula.

Number of posts	2	3	4	5
Number of rails	3	6	9	12

For 200 posts the number of rails = (200 – 1) x 3 = 597.

Formula: R = (p – 1) x 3

Problem 2

Pupils may want to use guess and check or organised list.

a. Using guess and check:
   10 of each coin = £1 + £5 = £6 too small
   20 of each coin = £2 + £10 = £12 too small
   30 of each coin = £3 + £15 = £18 correct solution
b. A similar approach or an organised table will also solve this problem. The solution is that 10 of each coin will make £3.50.
c. 50p and 20p. One of them is not a 50p - but the other one is!

Problem 3

Again pupils may guess or work systematically. Others may solve the problem as an equation.

2t + s = 1t + 3s, but t = 9
18 + s = 9 + 3s
9 = 2s
s = £4.50

Problem 4

Using an organised list and elimination of incorrect solutions would allow the problem to be solved.

0 1 2	b. The middle digit is prime	c. The last digit is a square no.
1 2 3	1 2 3	
2 3 4	2 3 4	**234 is the solution**
3 4 5		
4 5 6	4 5 6	
5 6 7		
6 7 8	6 7 8	
7 8 9		

Problem 5

An organised list will assist the pupils to solve the problem.

£5	£10	£20
6	0	0
4	1	0
2	2	0
2	0	1
0	3	0
0	1	1

Warm-up activity/Lesson starter

1 The sum of two consecutive numbers is 35. Find the numbers.

2 List the first five prime numbers.

3 Calculate $7^2 - 6^2$.

4 Find the tenth number in this number sequence:
2, 5, 8, 11…

Section 7

Problem solving tasks and lesson plans

Level F

Contents

Teachers Notes

Resources

Each lesson has:

1. lesson starter materials
2. pupil worksheets
3. teacher notes
4. solutions.

Teacher notes highlight the main strategies that could be used in the problems.

The lesson starters are designed, whenever possible, to be related to the problem that will follow by highlighting possible strategies that could be used in the lesson and also to review vocabulary used in some problems.

Overheads are included for some problems to clarify diagrams or to allow games to be played by the teacher with the class.

The order in which the problems are tackled is not important but it is suggested that Short and to the Point and Short and Testing could be tackled at the end as they require a variety of strategies to be used and could be used prior to pupils sitting their National Assessments.

Class structure

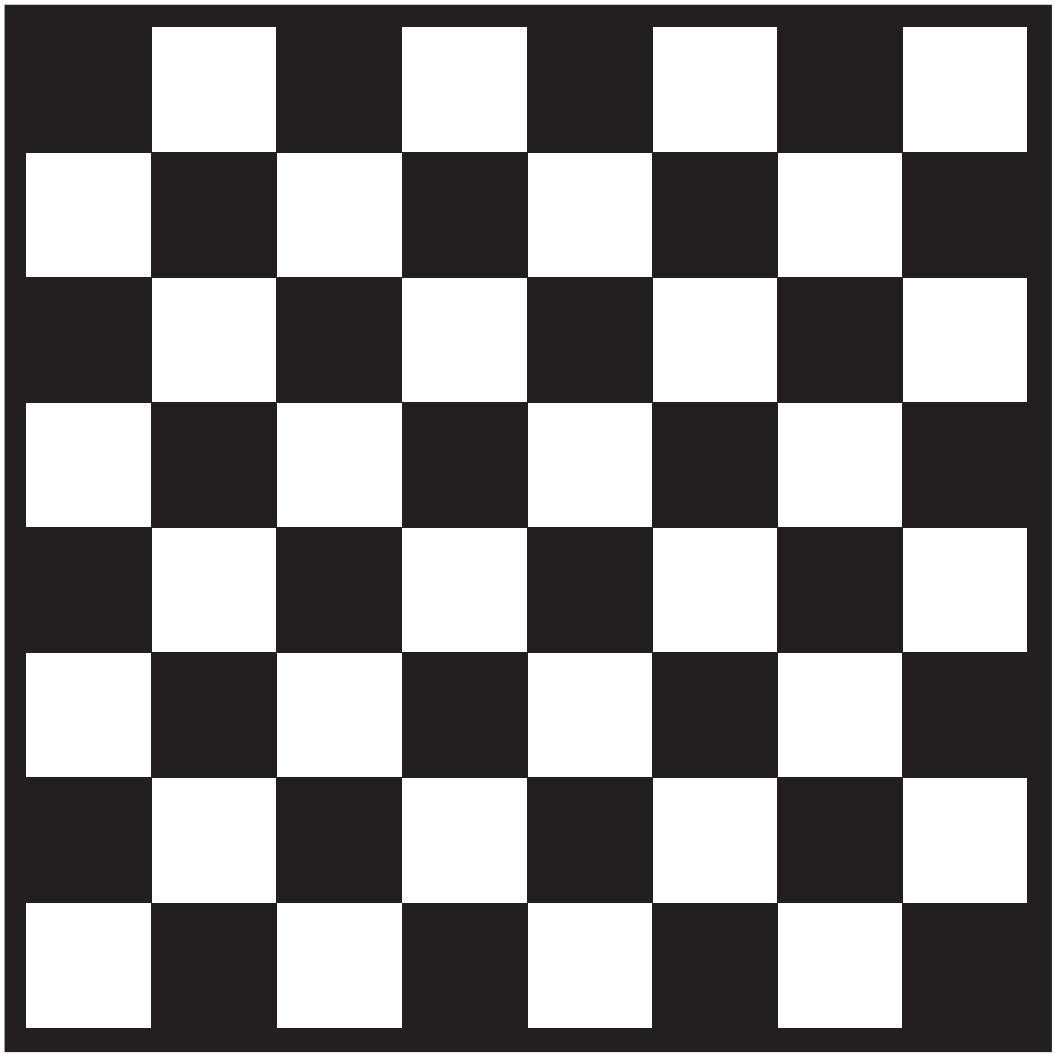
It is suggested that all of these problems should be tackled with the **pupils working in pairs or small groups.** Learning to reason requires interaction with someone else.

Pupils need to be challenged to think and talk about what they are doing when solving problems. It should be stated explicitly to the class why they are in pairs or groups. It is so that they can discuss the problem and so that they can support each other. It is not just by accident that they have a partner.

Problems are best solved by pupils discussing with each other the possible strategies that can be used to tackle the problem and by supporting each other through the solving process.

Most pupils will learn more and be more confident attempting the problems using a paired or group approach rather than asking them to solve the problems on their own.

Chessboard Challenge



Joe and Tilak are playing chess when Joe suggests that they count the number of squares on the chessboard.
'Easy,' says Tilak. 'There can only possibly be 64.'
'Definitely not. There are more than 100,' replies Joe.

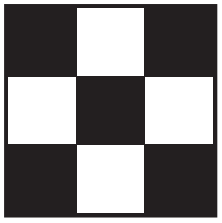
The problem

Find how many squares there are on a chessboard.
The squares can be of 1 by 1 or 2 by 2, etc.

Pupil Worksheet 1

Task 1

With a partner discuss the different size squares on the chessboard.
Write down all the different size squares in a chessboard.
For example, a 1 by 1 or 2 by 2, etc.



Task 2

With a partner, find the number of squares in a 3 rows by 3 columns chessboard.

Task 3

Discuss with your partner what strategies you could use to find out how many squares could be found in an 8 by 8 chessboard.
Write these down.

Task 4

Now try to tackle the original problem.

Extension

How many squares are there in a 10 by 10 chessboard?
Find a formula that you would use to calculate the number of squares in an n by n chessboard.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:

a. Try a simpler case
b. Be systematic
c. Draw a table
d. Look for a pattern
e. Use a formula.

RESOURCES The pupils will need squared paper.
Pupils have two worksheets that state the problem and outline the tasks.
A warm up/lesson starter activity sheet is included.

TIME 1–2 periods

LESSON STRUCTURE

c. Warm up (5 mins)
d. Introduce problem
e. Pupils discuss in pairs/groups and start (5–10 mins)
f. Teacher stops class and checks progress, intervenes if necessary (5 mins)
g. Problem attempted by pupils, teacher supports individual pupils (15 mins)
h. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils on their answers.
The activity is very much stand-alone and is designed to stimulate discussion.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
It is important that the pupils are fully aware of the different sizes of squares.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:

a. work systematically
b. record the results in an organised manner as they work through the problem.

Could the problem be made easier?
Would drawing a table help to spot a pattern?
The use of a formula could be highlighted through the extension.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION **Task 1**
1 by 1, 2 by 2, up to 8 by 8

Task 2
A 3 by 3 chessboard has:
One 3 by 3 square, four 2 by 2 squares, and nine 1 by 1 squares.
A total of 14.

Task 3
The task is designed to focus the pupils on the strategies required to solve the problem. The problem is much too difficult just to attempt to draw an 8 by 8 square and count all the squares.

Task 4
One method of solving the problem is to:
make it simpler by drawing smaller squares
draw a table
spot a pattern.

Dimensions of board	Number of squares	
1 by 1	1	
2 by 2	5	$1^2 + 2^2$
3 by 3	14	$1^2 + 2^2 + 3^2$
4 by 4	30	$1^2 + 2^2 + 3^2 + 4^2$
5 by 5	55	$1^2 + 2^2 + 3^2 + 4^2 + 5^2$
6 by 6	91	etc
7 by 7	140	
8 by 8	204	

Extension

An n by n square $= n^2 + (n - 1)^2 + (n - 2)^2 \ldots\ldots\ldots + 1^2$

10 x 10 $= 10^2 + 9^2 + 8^2 \ldots\ldots\ldots + 1^2$

Warm-up activity/Lesson starter

From the numbers 14, 15, 16, list the odd number out.

You must give a reason.

Try to find as many different reasons and different answers as possible.

Possible solutions

15 is the only odd number.

15 is the only product of 2 prime numbers.

16 is the only square number.

16 is the only multiple of 4.

14 is the only multiple of 7.

15 is the only multiple of 5.

16 is the only one that is a number after rotation by 180°, etc.

Crazy Towers

An architect is designing a new building. The building is to be in the shape of a tall tower. The architect decides that the tower will be a combination of dark marble and white marble or all black or all white marble.
Each level of the building will be either completely white or completely black.
A builder is asked to build the black and white tower. The builder is told the tower is to be 10 floors high, but not what combination of black and white the final design is to use.
The builder is confused and says 'There are lots of different combinations of black and white. Which one should I build?'

The Problem

How many different combinations of black and white or completely black or white are there for a tower with 10 floors?
One possible example of the tower is shown.

Pupil Worksheet

Task 1

With a partner discuss the problem, what it means and what possible problem solving strategies could be used to solve this problem.
Write down the strategies you will use to solve the problem.

Task 2

With a partner, attempt to solve the problem.

Task 3

Draw the design of tower you would choose if you were building the tower.

Extension

Find a formula which would allow you to calculate the different number of towers that could be built if there are n blocks.
Use this formula to calculate the different number of combinations of black and white if there are 15 blocks.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:

a. Try a simpler case
b. Organised list
c. Draw a table
d. Look for a pattern
e. Use a formula.

RESOURCES Pupils have two worksheets that state the problem and outline the tasks.
A warm up/lesson starter activity sheet is included.

TIME 1 period

LESSON STRUCTURE

f. Warm up (5 mins)
g. Introduce problem
h. Pupils discuss in pairs/groups and start (5–10 mins)
i. Teacher stops class and checks progress, intervenes if necessary (5 mins)
j. Problem attempted by pupils, teacher supports individual pupils (15 mins)
k. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils on how they obtained their answers.
There is also an example of indices in the warm up to assist the pupils with the problem.
Did any pupils make the questions simpler by changing the order of the additions?
Pupils should be reminded that making problems simpler is one method that can be used to solve difficult problems.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
It is important that the pupils are aware of how the tower is designed.
The class teacher should initially clarify the problem and **allow the pupils to formulate their own strategies for solving the problem.**
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.
Could the problem be made easier?
Would drawing a table help to spot a pattern?
The use of a formula could be highlighted through the extension.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION **Task 1**
Some possible strategies could include organised lists, make a table, spot a pattern, and use a formula.

Task 2
A 1-block tower would have either a black (b) or white (w) block = 2 combinations.
A 2-block = ww, wb, bw, bb = 4 combinations.
A 3-block = www, wwb, wbw, bww, bbw, bwb, wbb, bbb = 8 combinations, etc.
Drawing a table now assists the spotting of a pattern.

No. of blocks	1	2	3	4	5	6	7	8	9	10
No. of combinations	2	4	8	16	32	64	128	256	512	1024

Extension

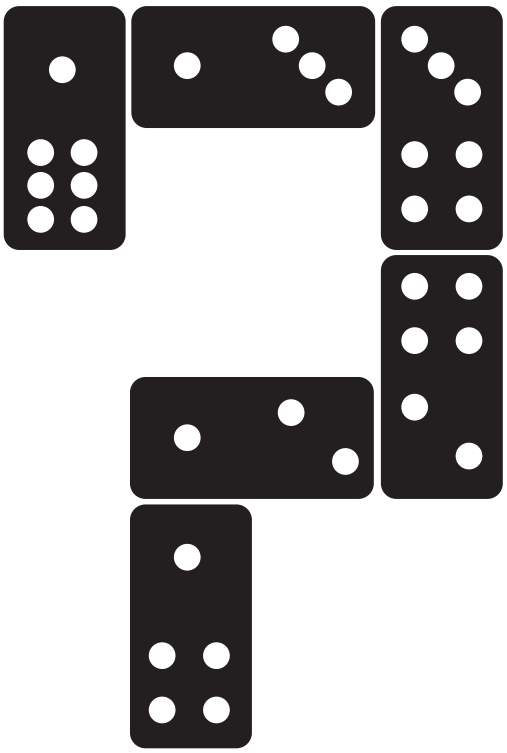
No. of combinations for n blocks = 2^n
15 blocks = 2^{15}

Warm-up activity/Lesson starter

10.75	6.51	10^4
22.5	1.9	3^3
$15\frac{1}{4}$	21	972
150%	3.49	1^1

1. Add the numbers in the first column.
2. Add the numbers in the middle column.
3. Add the numbers in the end column.

Deca-dominoes



Kurt, Ali and Mary enjoy playing dominoes and decide that it would be fun if all the class could play the same game of dominoes at the same time.
Ali suggests that each person in the class must have at least 2 dominoes to make the game fair.
Mary insists that they would not have enough dominoes for the class of 28 pupils to have at least two each. John says that they will have to make a special set of dominoes and that this new set of dominoes should be numbered 0 to 9.

Problem

Would a set of dominoes numbered zero to nine be sufficient to allow 28 pupils two dominoes each?

Pupil Worksheet 1

Task 1

With a partner write down the numbers used in a normal domino set.
Remember to include zero for blanks.
Discuss with your partner the rules of dominoes.
What dominoes are the same?
How do you start?
What dominoes can be played next?
What does 'chapping' mean in dominoes?
How do you win a game of dominoes?

Task 2

With a partner list all the different dominoes you would have if you only used the digits 0, 1, 2 and 3.

Task 3

Discuss with your partner what strategies you could use to find out how many dominoes would make up a set numbered 0 to 9.
Now attempt the problem. How many dominoes would be in a set which is numbered 0 to 9?

Extension

Find a formula which would allow you to calculate the number of dominoes in a set numbered 0 to 99.
With your partner make the set of dominoes for the digits 0, 1, 2, 3.
With your partner use the dominoes you have made to play some games of dominoes.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:

a. Try a simpler case
b. Be systematic
c. Draw a table
d. Look for a pattern
e. Use a formula.

RESOURCES Pupils have two worksheets that state the problem and outline the tasks.
Pupils will need card and scissors to attempt the task.
A warm up/lesson starter activity sheet is included.

TIME 1–2 periods

LESSON STRUCTURE

a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils on their answers.
The activity is very much stand-alone and is designed to stimulate discussion.

STARTING Read through the problem with the pupils.
Please note that some pupils will be unaware of what constitutes a normal domino set, and time should be taken to discuss this before embarking on the problem.
It is essential that pupils realise that some dominoes are equivalent.

For example, 

these are the same and should only be counted once:
Ask them to express the problem in their own words.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
work systematically
record the results in an organised manner as they work through the problem.
A discussion on the best way to record the results and the use of tables to spot patterns would be valuable to many pupils.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be asked to explain their solutions.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION Task 1

0, 1, 2, 3, 4, 5, 6

Task 2

0, 0	1, 1	2, 2	3, 3
0, 1	1, 2	2, 3	
0, 2	1, 3		
0, 3			

Task 3

Method 1

Pupils may list all the dominoes which make up different sets.

1 digit	2 digit	3 digit	4digit
0, 0	0, 0	0, 0	0, 0
1	0, 1	0, 1	0, 1
	1, 1	0, 2	0, 2
	3	1, 1	0, 3
		1, 2	1, 1
		2, 2	1, 2
		6	1, 3
			2, 2
			2, 3
			3, 3
			10

Pupils may be able to spot a pattern now or some will want to continue listing all the possible domino permutations. However continuing to list all dominoes up to the 10 digits will be very time consuming.

5 digit	6 digit	7digit	8 digit	9 digit	10 digit
10 + 5 = 15	15 + 6 = 21	21 + 7 = 28	28 + 8 = 36	36 + 9 = 45	45 + 10 = 55

The solution to the problem is that there are not enough dominoes for a class of 28 to have 2 dominoes each. There would be 1 domino short.

METHOD 2 As method 1 but use a table to find a formula:

No. of digits	No. of dominoes	
1	1	(1 x 2) / 2
2	3	(2 x 3) / 2
3	6	(3 x 4) / 2
4	10	(4 x 5) / 2, etc

10 = (10 x 11) / 2 = 55

Other methods can be used to solve this problem.

For example, the sum of the digits 1, 2, 3……10 will also give the solution.

EXTENSION Formula:

For n digits the number of dominoes = n (n + 1) / 2

For 100 digits, the number of dominoes = 100 (101) / 2 = 5050

Warm-up activity/Lesson starter

From the numbers 9, 10, 11, list the odd number out.
You must give a reason.
Try to find as many different reasons as possible.

Possible solutions

10 is the only even number.
11 is the only prime number.
10 is the only multiple of 5.
9 is the only squared number.
9 is the only multiple of 3.
11 is the only palindrome.
10 is the only product of two distinctive primes.
10 is the only one that is not a number after rotation by 180°

Football Crazy

Billy, Inga, Ali and Roslyn are discussing ways to raise money for charity. Inga suggests that they run a football tournament for all the classes in the school.
Ali says 'With 30 classes in the school, why don't we have a league when every team plays each other twice?'
'This will take longer than the school year,' replies Roslyn.
'I don't think so. Let's work it out,' says Billy.

Problem

There are 30 teams, each game last 10 minutes and each team play each other twice. Lunchtime lasts 1 hour. How many lunchtimes does it take to play the tournament?

Task 1

With a partner discuss the problem and what strategies could be used to tackle it.

Task 2

Aberdeen, Celtic, Rangers and Dundee all play each other twice in a mini-tournament. List all the games that would be played.

Task 3

Hearts join the four teams above in a new tournament. How many games would be played now? List all the games. Remember all teams play each other twice.

Task 4

Go back to the original problem. Discuss it again with your partner. What will be the best strategy to solve it?
Now attempt to solve the problem.
How many lunchtimes would it take to play the tournament?

Extension

Find a formula which would allow you to calculate the number of games in a tournament with n teams, when each team plays each other twice.
The head teacher decides that only 100 lunchtimes can be used for the football tournament.
How many teams could take part in the football tournament now?

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:

a. Try a simpler case
b. Be systematic
c. Look for a pattern
d. Use a formula.

RESOURCES Pupils have two worksheets that state the problem and outline the tasks.
A warm up/lesson starter activity sheet is included.

TIME 1–2 periods

LESSON STRUCTURE

a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils on the answers.
The activity is very much stand-alone and is designed to stimulate discussion.

STARTING Read through the problem with the pupils.
Ensure that the pupils understand the concept of playing each team twice.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using. Can they start to tackle the problem?
Is further discussion required to support the pupils?

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:

a. work systematically
b. record the results in an organised manner as they work through the problem.

A discussion on the best way to record the results and the use of tables to spot patterns would be valuable to many pupils.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be encouraged to explain their solutions to the rest of the class, paying particular reference to the strategies they have used.

SOLUTION **Task 2**

A v C	C v A	R v A	D v A
A v D	C v D	R v D	D v C
A v R	C v R	R v C	D v R

Task 3

Pupils may wish to list the games or spot a pattern.

A v C	C v A	R v A	D v A	H v A
A v D	C v D	R v D	D v C	H v C
A v R	C v R	R v C	D v R	H v D
A v H	C v H	R v H	D v H	H v R

A total of 20 games

Task 4

A table to spot a pattern is one of the strategies that may be used to solve the problem.

No. of teams	1	2	3	4	5
No. of games	0	2	6	12	20

Pupils may wish to continue the table using the differences in the number of games, e.g. + 2, + 4, + 6, + 8, etc.

Pupils could also solve the problem by spotting a pattern.

No. of teams	No. of games
1	0 = 1 x 0
2	2 = 2 x 1
3	6 = 3 x 2
4	12 = 4 x 3
5	20 = 5 x 4
6	30 = 6 x 5

So for 30 teams the number of games = 30 x 29 = 870 games.

Time = 870 x 10 = 8700 minutes = 145 hours = 145 lunch times.

This is less than a school year.

EXTENSION For n teams the number games can be found by the formula:

Number of games = $n (n - 1) = n^2 - n$.

25 teams would require 600 games, which would require 100 lunch times.

Warm-up activity/Lesson starter

From the numbers 15, 16 and 17, list the odd number out.
You must give a reason.
Try to find as many different reasons as possible.

Possible solutions

16 is the only even number.
17 is the only prime number.
15 is the only multiple of 5.
16 is the only squared number.
15 is the only multiple of 3.
15 is the only product of two distinctive primes.
16 is the only number that can be expressed as a power of 2.

New Year's Eve

It is New Year's Eve, and the McDonald family have all met up for a celebration. Traditionally, at exactly midnight, all family members shake hands with all the other family members. There are 30 members of the McDonald family at the celebration this year.

Flora, the teenage daughter, is desperate to leave the family celebration to go out partying with her friends and wonders how long it is going to take to shake all the hands. She knows she cannot leave until everyone has finished shaking hands.

She estimates that each handshake will take 4 seconds.

Problem

How many handshakes will be needed for all the family to shake hands once with each other?

How many seconds will it take for all of the McDonald family to shake hands?

What time will it be when Flora finishes shaking hands with the whole family?

Pupil Worksheet 1

Task 1

In groups of 4 or 5 or 6 shake hands with everyone else.

Do not shake hands with the same person twice.

Record how many handshakes are required.

The group will have to organise carefully how this is carried out so that the results are correct.

Task 2

Check your result by doing task 1 again.

Task 3

Discuss with your group what strategies could be used to solve the problem.

Write these strategies down.

Task 4

Attempt to solve the problem. How many handshakes will be needed? How many seconds will this take?

Is it possible to work out what time it will be when the McDonald family have finished shaking hands?

Extension

Find a formula which would allow you to calculate the number of handshakes for n people.

Use this formula to find the number of handshakes required for a family of 100.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:

a. Act it out
b. Draw a table
c. Look for a pattern.

RESOURCES Pupils have two worksheets that state the problem and clarify the tasks.
A warm up/lesson starter activity sheet is included.

TIME 1–2 periods

LESSON STRUCTURE

a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in groups and start (10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils on their answers.
The activity is designed to review the pupils' knowledge of indices before beginning the problem.
The formula that solves the problem involves indices.

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
It is suggested that they discuss and tackle the problems in groups to allow them to act out the problem.
The problem can be solved without acting out, but by allowing the pupils to act it will deepen their understanding of the problem and allow them to share ideas on how to solve the problem.

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:

a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be encouraged to explain their solutions to the rest of the class, paying particular reference to the strategies they have used.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION **Task 1**

A group of 4 will require 6 handshakes.
A group of 5 will require 10 handshakes.
A group of 6 will require 15 handshakes.

Task 3

The strategies could include act it out, draw a table, spot a pattern.

Task 4

No. of people	2	3	4	5	6
No. of handshakes	1	2	6	10	15

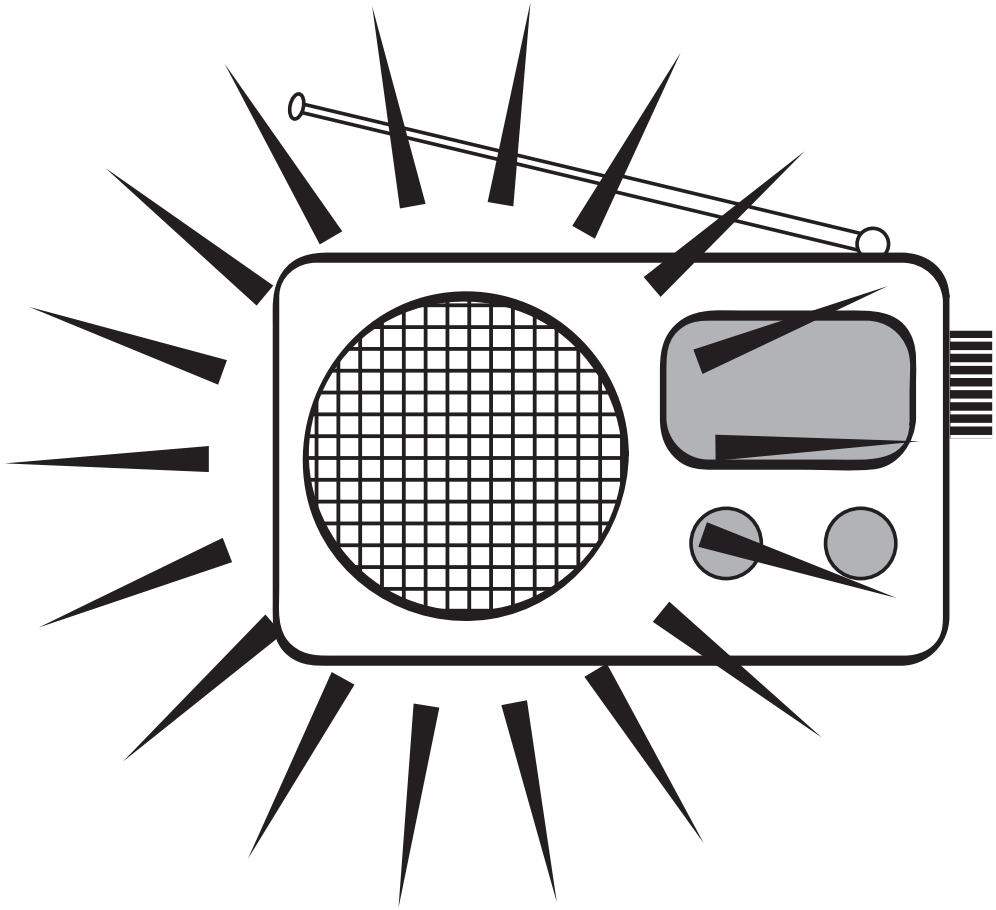
The pattern can be continued to find the solution to the problem.
For 30 people 435 handshakes are required.
The number of seconds = 435 x 4 =1740
The number of minutes = 1740 / 60 = 29 minutes
Time 12.29 am or 0029
Since the family will shake hands simultaneously the actual time taken will be less than 29 minutes.

EXTENSION The formula for n people is $\frac{1}{2}$ n (n – 1)
For n = 100, the number of handshakes = $\frac{1}{2}$ x100 x 99 = 4950

Warm-up activity/Lesson starter

1. $12^2 + 3^3 =$
2. $4^3 =$
   $4^2 =$
   $4^1 =$
   $4^0 =$

Radio Ga-Ga



A local radio station is running a competition. To win you must phone in the correct answer to their question. But only the first one hundred callers receive a prize.
The prize received is money.
The amount of money depends on the sum of the even numbers.
For example, if you are the:
first caller you will receive £2 = £2
second caller = £2 + £4 = £6
third caller = £2 + £4 + £6 = £12.

Problem

If you are the 100th caller how much money will you win?

Pupil Worksheet

Task 1

With a partner write down the first 14 even numbers.

Note: Do not count zero as an even number.

Task 2

With a partner discuss the best way to add up the first 14 even numbers.

Task 3

With your partner calculate the prize money received if you are the 14th caller.

Task 4

With a partner discuss the best way to add up the first 20 even numbers.

Calculate the prize money for the 20th caller.

Task 5

Discuss with your partner the strategies you could use to find the amount of prize money received by the 100th caller.

Now attempt to solve the problem.

Extension

Find a formula, which would allow you to calculate the sum of any number of even numbers.

Use this formula to calculate the sum of the first 1,000 even numbers.

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:

a. Try a simpler case
b. Be systematic
c. Draw a table
d. Look for a pattern
e. Use a formula.

RESOURCES Pupils have two worksheets that state the problem and clarify the tasks.
A warm up/lesson starter activity sheet is included.

TIME 1 period

LESSON STRUCTURE

f. Warm up (5 mins)
g. Introduce problem
h. Pupils discuss in pairs/groups and start (5–10 mins)
i. Teacher stops class and checks progress, intervenes if necessary (5 mins)
j. Problem attempted by pupils, teacher supports individual pupils (15 mins)
k. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils on how they obtained their answers.
Did any pupils make the questions simpler by changing the order of the additions?
Pupils should be reminded that making problems simpler is one method that can be used to solve difficult problems.

STARTING Read through the problem with the pupils.
Please note that the pupils should not include zero.
Ask them to express the problem in their own words.
It is suggested that they discuss the problems in pairs before tackling the worksheet.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using.

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:

a. work systematically
b. record the results in an organised manner as they work through the problem.

A discussion on the best way to record the results and the use of tables to spot patterns would be valuable to many pupils.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be encouraged to explain their solutions to the rest of the class, paying particular reference to the strategies they have used.
If they were to tackle a similar problem again, which strategies would they use?
Could they improve on their strategies?

SOLUTION **Task 1**

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28

Task 3

2 + 28 = 30
4 + 26 = 30
6 + 24 = 30, etc
Total = 7 x 30 = 210

Task 4

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40
2 + 40 = 42
4 + 38 = 42
6 + 36 = 42
Total is 10 x 42 = 420

Task 5

Method 1
First 100 even numbers are: 2, 4, 6, 8……192, 194, 196, 200.
Sum = 2 + 200, 4 + 198, 6 + 196, 8 + 194, etc
Total = 202 x 50 = 10,100
Solution is 10,100.

Method 2

Even numbers	Sum of even numbers	
1st	2	= 2 x 1
2nd	= 2 + 4 = 6	= 3 x 2
3rd	= 2 + 4 + 6 = 12	= 4 x 3
4th	= 2 + 4 + 6 + 8 = 20	= 5 x 4
100th	= 101 x 100 = 10,100	

EXTENSION

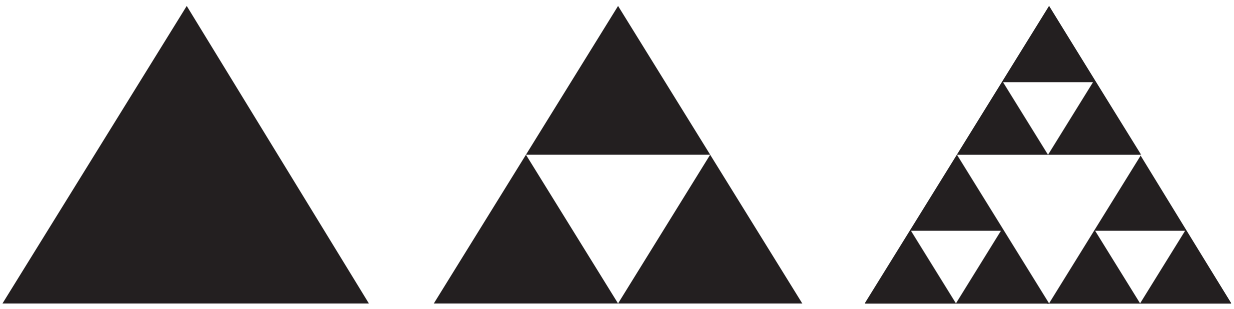
nth	= (n + 1) n = n^2 + n	
1000th	= (1001) 1000 =1,001,000	

Warm-up activity/Lesson starter

3301	6.55	1 and $^1/_2$
3500	1.9	1.75
3700	7.1	225%
4000	3.45	2.5

1. Find the sum of the numbers in the first column.
2. Add the numbers in the end column.
3. Find the sum of all the even numbers in the table.

Triangular Puzzles



These figures illustrate a design known as the Sierpinski triangle.

Problem

To find formulae for the number of black and white triangles in the nth figure.

Pupil Worksheet

Task 1

Look carefully at the number of black triangles in the pattern.

Discuss with your partner how the pattern is built up.

Task 2

Copy and complete the table below:

Figure	No. of Black Triangles
1	1
2	3
3	
4	
5	
6	
10	
n	

Task 3

Look carefully at the number of white triangles in the pattern.

Discuss with your partner how the pattern is built up.

Consider how the white triangles are made from the black triangles.

Task 4

Copy and complete the table below:

Figure	No. of White Triangles
1	0
2	1
3	
4	
5	
6	
10	
n	

Teachers Notes

AIMS To engage pupils in the following problem solving strategies:

a. Draw a table
b. Look for a pattern
c. Use a formula.

RESOURCES Pupils have two worksheets that state the problem, outline the tasks and clarify the problem.
A warm up/lesson starter activity sheet is included.

TIME 1 period

LESSON STRUCTURE

a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils on their answers.
The activity is designed to review the pupils' knowledge of indices before beginning the problem.
The formula that solves the problem involves indices.
It is essential that the pupils know $x^0 = 1$.

STARTING Read through the problem with the pupils.
Pupils will struggle with the problem unless they aware of indices.
Ask them to express the problem in their own words.
It is suggested that they discuss and tackle the problem in pairs.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using.
Further guidance may be required if pupils are still finding difficulty, but they should be allowed some time to struggle with the problem before they are directed to a strategy that allows them to try to solve the problem.

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:

a. work systematically

b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used. Pupils should be encouraged to explain their solutions to the rest of the class, paying particular reference to the strategies they have used.

SOLUTION **Task 2**

Figure	No. of Black Triangles
1	1
2	3
3	9
4	27
5	81
6	243
10	3^9
n	3^{n-1}

Task 3

Figure	No. of White Triangles
1	0
2	1
3	3
4	9
5	27
6	81
10	3^8
n	3^{n-2}

Warm-up activity/Lesson starter

The answer is 180

Find five questions

Suitable Marriages

Anna and Hazim are discussing star signs and decide that:

a. marriages should not take place between people of the same star signs

b. marriages should not take place between people who have star signs that are next to each other in the calendar

c. all other combinations of star signs would make suitable marriages.

Problem

How many different combinations of star signs would make a suitable marriage?

Pupil Worksheet

Task 1

With a partner write down as many star signs as you know.

Task 2

Discuss with your partner which of the following marriages would be suitable.
The fact sheet on the next page will help.
Remember that marriages of people with the same star sign and marriages of adjacent star signs are not suitable.

Cancer and Leo
Scorpio and Libra
Leo and Pisces
Leo and Leo
Sagittarius and Leo

Task 3

Discuss with your partner the strategies you will use to find how many combinations of the 12 star signs would make a suitable marriage.
Write these strategies down.

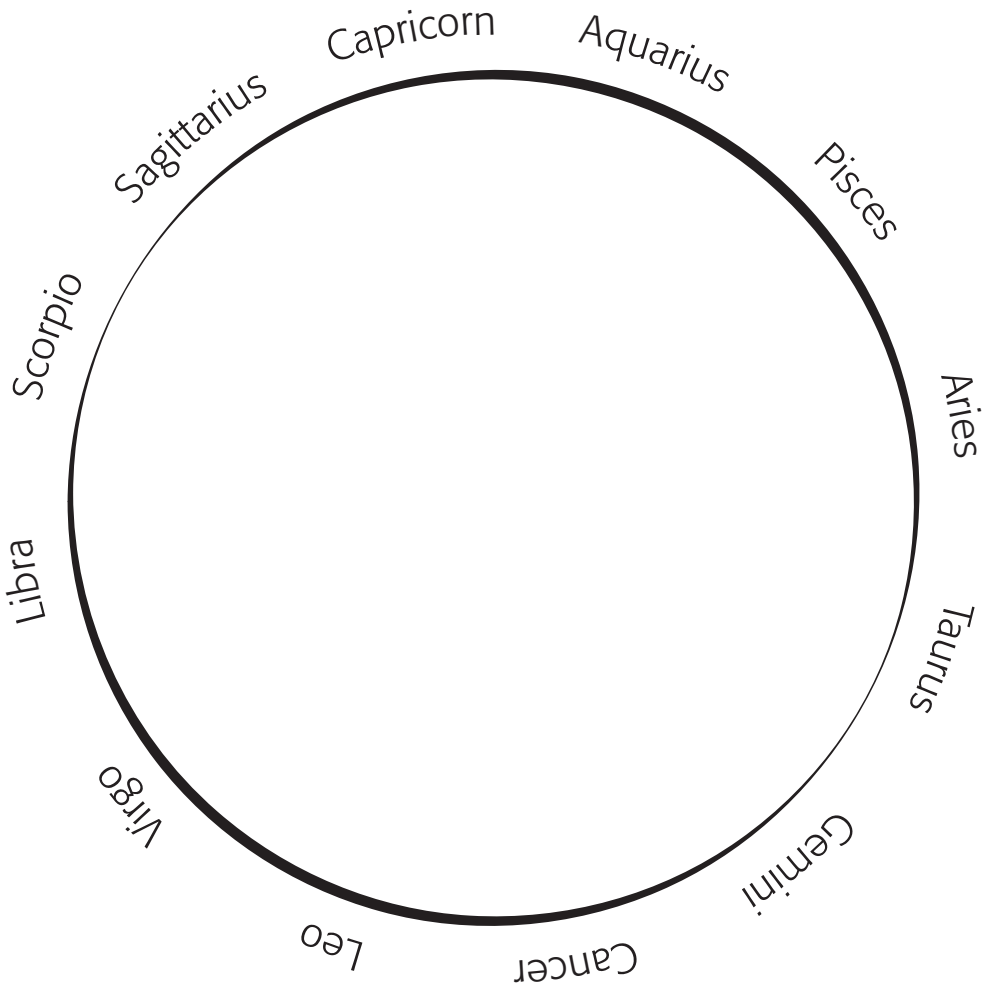
Task 4

Now attempt to solve the problem.
It is important that you record your work neatly as you go through the problem.

Extension

Find a formula to calculate the number of suitable marriages.
Now use the formula to calculate the number of suitable marriages if there were 100 different star signs.

Fact sheet


Capricorn
Aquarius
Pisces
Aries
Taurus
Gemini
Cancer
Leo
Virgo
Libra
Scorpio
Sagittarius

Teachers Notes

AIMS To give pupils opportunities to use:

a. Drawing a diagram
b. Try a simpler case
c. Look for a pattern
d. Make and test a conjecture.

RESOURCES Pupils have three worksheets that state the problem and outline the tasks.
An overhead transparency/pupil worksheet is provided to assist the introduction of the problem and to clarify which marriages are suitable.

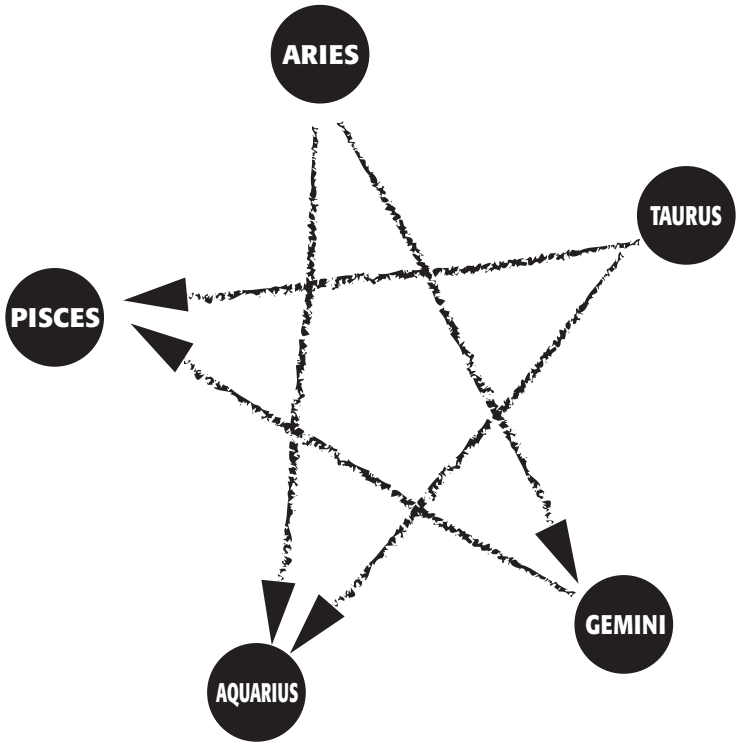
TIME 1 to 2 periods

LESSON STRUCTURE

a. Warm up (5 mins)
b. Introduce problem
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problem attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

STARTING Read through the problem with the pupils.
Ask them to express the problem in their own words.
Ensure that pupils understand the constrictions placed on a suitable marriage.
However, teachers should not suggest possible strategies to solve the problem.
Use the overhead transparency/pupil worksheet with the 3, 4, 5 and 6 star signs to support the pupils if required.

TASK 2 Encourage a systematic approach to completing a diagram, e.g. starting with one star sign and working around the diagram clockwise, recording the pairs as they are found.



Aries could marry Aquarius or Gemini: 2 pairs.
Taurus could marry Pisces or Aquarius: 2 pairs.
Gemini could marry Aries, but this has already been counted, or Pisces: 1 pair.
Aquarius with Aries or Taurus have been counted.
Pisces with Taurus or Gemini have been counted.
Number of suitable marriages = 2 + 2 + 1 = 5.

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results as they work through the problem.
A discussion on the best way to record the results and the use of tables to spot patterns would be valuable to many pupils.
Can the results they have produced allow them to predict a solution to the problem?
If they have made a conjecture how can they check that the solution is correct?

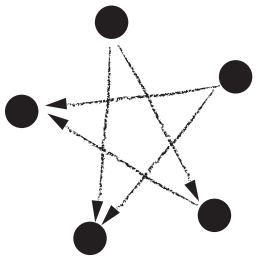
SOLUTION There are various ways to solve the problem. It is important not to be too prescriptive in leading the pupils in their thinking. Allow them the freedom to think creatively and make their own mistakes, and welcome original thinking even if it is not the most effective method.
The lessons should finish with a discussion on what the pupils regard as the most effective method.

Method 1

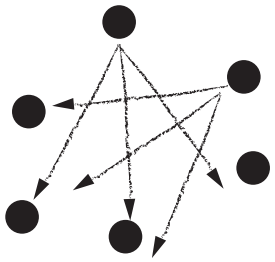
From a diagram the pupils may realise that if we have 12 star signs, each sign can be paired with 9 others, not with themselves, and not with the 2 signs adjacent to them.
So that makes:
12 X 9 = 108
But in doing this method we are counting each pair twice.
Total number of pairs = 108 / 2 = 54

Method 2

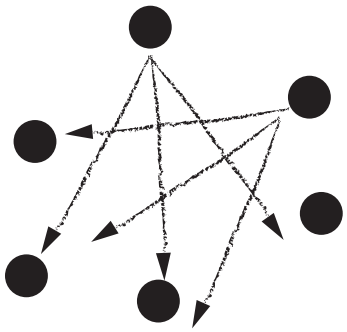
From a diagram:



No. of pairs = 2 + 2 + 1 = 5



No. of pairs = 3 + 3+ 2 + 1 = 9



No. of pairs = 4 + 4 + 3 + 2 + 1 = 14

Continue to draw diagrams until a pattern can be spotted.

A table will help to spot the pattern.

Number of star signs	Number of marriages
3	0
4	1 + 1= 2
5	2 + 2 + 1 = 5
6	3 + 3 + 2 + 1 = 9
7	4 + 4 + 3 + 2 + 1 = 14
8	5 + 5 + 4 + 3 + 2 + 1 = 20
12 signs =	9 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 54

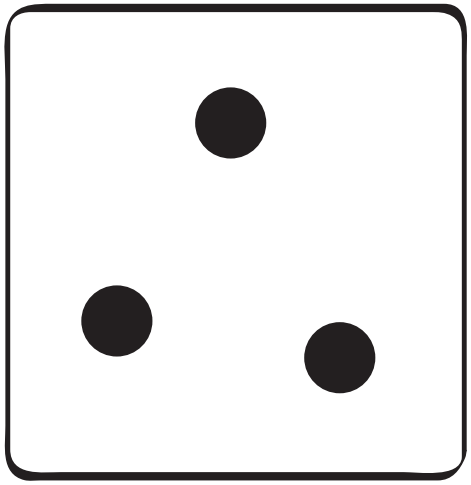
EXTENSION For n star signs the number of suitable marriages would be

$^1/_2$ x n (n – 3)

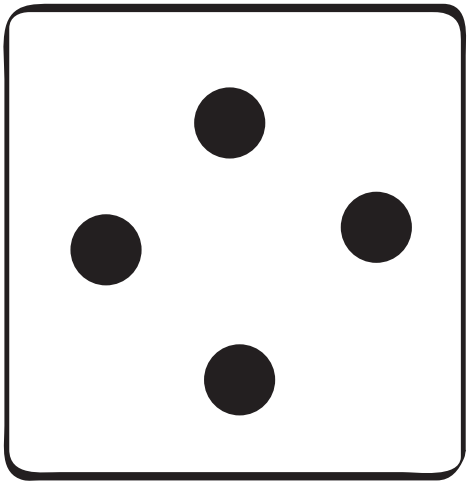
$$100 \text{ star signs} = \frac{100\ (97)}{2} = 4850$$

Overhead Transparency

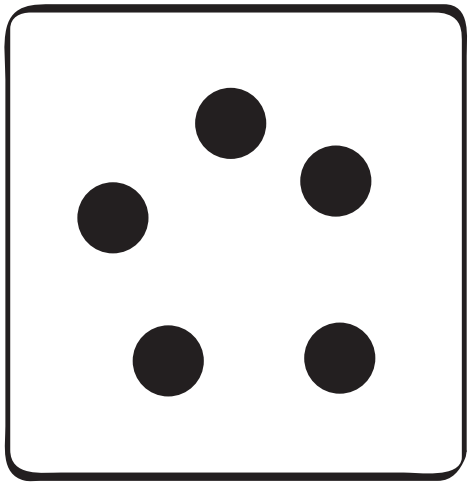
Three Star Signs



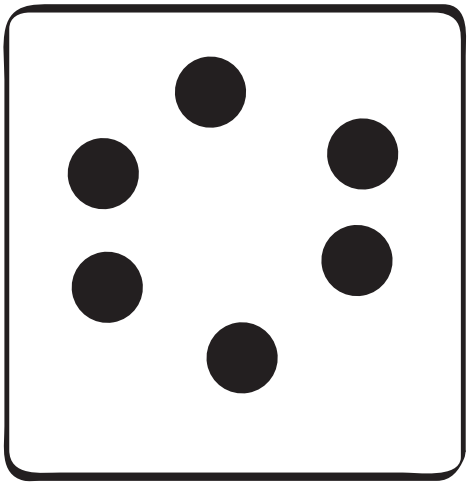
Four Star Signs



Five Star Signs



Six Star Signs



Short and Testing

For each problem discuss with your partner:

- what the problem means
- what strategies you could use to tackle the problems
- what way is best to start the problems.

Then attempt to solve the problems.

Task 1

Radjeep had some money. He spent £20. He then spent a quarter of what was left. He has now spent another £5 and has £40 left.

How much did he start with?

Task 2

In a bag of sweets there are 20 strawberry creams and 25 orange creams. Five eighths of the rest of the sweets are toffees and the remainder are coffee flavoured.

If there are 21 coffee flavoured sweets, how many sweets are in the bag in total?

Task 3

Make these sums correct by using the following numbers only once:

250%, $1\frac{3}{4}$, $\sqrt{6400}$, 1.55, 4^3, 7.45, 3^2

1	100 x 0.8	=	……
2	…… of 3.6	=	9
3	24 x ……	=	42
4.	2^6	=	……
5.	……	=	……

Task 4

The conversation on the next page is between two friends who shared a room together at college when they were younger and who meet up again many years later. Use the clues to find the ages of the children.


I have 3 children
How old are they?
The product of their ages is 36
I need more information
The sum of their ages is our old room number
There is still not enough information
The oldest one is a girl
Now I know their ages

Teachers Notes

AIMS To engage the pupils in a number of problem solving strategies.

RESOURCES Pupils have three worksheets that state the problem, outline the tasks and clarify the problem.
A warm up is included.

TIME 1 to 2 periods

LESSON STRUCTURE a. Warm up (5 mins)
b. Introduce the problems
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problems attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problem (5–10 mins).

WARM UP To remind the pupils of squares, powers, square root, etc

STARTING Read through the problems with the pupils.
It may be better to deal with one problem at a time.
Pupils will struggle with problem 3 unless they are aware of indices.
Ask them to express the problem in their own words.
It is suggested that they discuss and tackle the problems in pairs.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using.
Further guidance may be required if pupils are still finding difficulty, but they should be allowed some time to struggle with the problems before they are directed to a strategy that allows them to try to solve the problem.

DOING Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION Pupils should now go on to solve the problem using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

Solutions **Task 1**

Guess and check, and work backwards would be methods to solve this problem, or an equation could be formed:

$(x - 20) - {}^{1}/_{4}(x - 20) - 5 = 40$

$x = £80$

Task 2

3/8 of the rest are coffee = 21

So 1/8 = 7

The number of sweets = 56 + 20 + 25 = 101

Task 3

$100 \times 0.8 = \sqrt{6400}$

250% of 3.6 = 9

$24 \times 1{}^{3}/_{4} = 42$

26 = 43

A variety of solutions are possible:

$3^2 - 1.55 = 7.45$, or $7.45 + 1.55 = 3^2$, or $3^2 - 7.45 = 1.55$

Task 4

The problem can be solved by using an organised list and eliminating incorrect solutions and logical deductions.

The product of their ages is 36. The sum of their ages is our old room number

So the ages could be:

1, 1, 36	1 + 1 + 36 = 38
1, 2, 18	1 + 2 + 18 = 21
1, 3, 12	1 + 3 + 12 = 16
1, 4, 9	1 + 4 + 9 = 14
1, 6, 6	1 + 6 + 6 = 13
2, 2, 9	2 + 2 + 9 = 13
2, 3, 6	2 + 3 + 6 = 11
3, 3, 4	3 + 3 + 4 = 10

We can assume that the friend knows the room number, so they should be able to identify the ages from the sum of the products unless more than one of the sums give the correct room number.

1 + 6 + 6 = 13, and

2 + 2 + 9 = 13

have the same sum.

The oldest is a girl tells us that the largest number must be unique.

The ages of the children must be 2, 2 and 9.

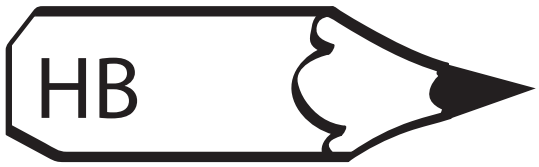
Warm-up activity/Lesson starter

Re-arrange these numbers from the smallest to the largest:

4^3 $\sqrt{2500}$ 7^2 2^7 1^{10}

Short and to the Point

Pupil Worksheet



For each problem discuss with your partner:

- what the problem means
- what strategies you could use to tackle the problems
- what way is best to start the problems.

Then attempt to solve the problems.

Task 1

In a theatre the seats are in rows of increasing size.

The size of the rows increases using the following pattern:

- the first row has 7 seats
- the second 10 seats
- the third 12 seats
- the fourth 15 seats and the fifth 17 seats.

How many seats will be in the 12th row?

Task 2

a. What would the 15th number be in this pattern: 1, 4, 9, 16?

b. What would the 6th number be in this pattern: 1, 8, 27, 64?

c. What would the next two numbers be in this sequence: 27, 9, 3, 1?

Task 3

In a maths quiz game the teacher awards points depending on the difficulty of the question.

- A question = 10 points
- B question = 5 points
- C question = 2 points

Joanne scored 29 points in the quiz.

Copy and complete the table, showing different ways to obtain 29 points.

There is no limit to the number of questions that could be asked.

10 points (A)	5 points (B)	2 points (C)	Total
2	1	2	29

Task 4

Mark receives £37 change from a shop.

He asked the shop assistant if he could receive at least one £10 note and at least one £5 note and at least one £1 coin.

List all the different ways Mark can obtain his change using only £10 notes, £5 notes and £1 coins.

Task 5

At Inverness High School 3/5 of the pupils support Inverness Caley Thistle (ICT), and the rest support Aberdeen.

At away matches 1/6 of the pupils who support ICT attend the match.

At the last away match 18 pupils attended to support ICT.

How many pupils support Aberdeen?

Teachers Notes

AIMS To engage pupils in a variety of problems which are similar to the problems that are asked in the National Tests.

RESOURCES Pupils have two worksheets that state the problem, outline the tasks and clarify the problem.
A warm up is included.

TIME 1 period

LESSON STRUCTURE a. Warm up (5 mins)
b. Introduce problems
c. Pupils discuss in pairs/groups and start (5–10 mins)
d. Teacher stops class and checks progress, intervenes if necessary (5 mins)
e. Problems attempted by pupils, teacher supports individual pupils (15 mins)
f. Plenary – discussing the main strategies used in the problems (5–10 mins).

WARM UP Allow the pupils to tackle the warm-up activity and then question the pupils on their answers.
The activity is not directly connected to the problems.

STARTING Read through the problems with the pupils.
It may be easier to tackle the problems one at a time.
Ask them to express the problems in their own words.
It is suggested that they discuss and tackle the problems in pairs.
After allowing the pupils to struggle with the problem for, say, 10 minutes, the class should be brought together to discuss what progress has been made and what strategies they are using.
Further guidance may be required if pupils are still finding difficulty, but they should be allowed some time to struggle with the problems before they are directed to a strategy that could allow them to solve the problems.

DOING Pupils should now go on to solve the problems using whichever strategy they think is appropriate. They should be encouraged to:
a. work systematically
b. record the results in an organised manner as they work through the problem.

PLENARY SESSION The plenary sessions should be used to highlight the methods used.
Pupils should be encouraged to explain their solutions to the rest of the class, paying particular reference to the strategies they have used.

Solutions **Task 1**

7, 10, 12, 15, 17, 20, 22, 25, 27, 30, 32, 35

There are 35 seats in the 12th row.

Task 2

1, 4, 9, 16, … 15th term = $15^2 = 225$

1, 8, 27, 64 … 6th term = $6^3 = 216$

27, 9, 3, 1… 1/3, 1/9

Task 3

10 points (A)	5 points (B)	2 points (C)	Total
2	1	2	29
1	3	2	29
1	1	7	29
0	5	2	29
0	3	7	29
0	1	12	29

Task 4

£10	£5	£1
1	1	22
1	2	17
1	3	12
1	4	7
1	5	2
2	1	12
2	2	7
2	3	2
3	1	2

Task 5

3/5 of 1/6 = 3/30 = 18

Whole school = 180

Aberdeen fans = 2/5 of 180 = 72

Warm-up activity/Lesson starter

1. $14^2 + 1^3$

2. $5^3 =$
   $5^2 =$
   $5^1 =$
   $5^0 =$

3. $\sqrt{169 - 2^4}$

Section 8

If stuck.....

If stuck….

- Talk to your partner about the problem.
- What is the problem asking you to find out?
- What do you know about the problem?
- What strategies might help?
- Have we tried a similar problem before?
- Can we make the problem simpler?
- Would a diagram help?
- Can we guess and check?
- What can I do to start?

If Stuck....

Talk to your partner about the problem.

What is the problem asking you to find out?

What do you know about the problem?

What strategies might help?

Have we tried a similar problem before?

Can we make the problem simpler?

Would a diagram help?

Can we guess and check?

What can I do to start?