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Bayla's Bracelets - March 3, 2014

Problems & Solutions

Bayla enjoys using a special loom and bands to make bracelets for herself and for her friends. She has bands in 10 different colors. Bayla wants to make a bracelet that has bands in only 7 different colors. How many different combinations of 7 colors can Bayla use to make a bracelet?

We are asked to determine how many combinations there are of 7 colors given 10 colors from which to choose. There are $_{10}\text{C}_7 = 10!/(3! \times 7!) = (10 \times 9 \times 8)/(3 \times 2 \times 1) = 720/6 = 120$ different combinations of 7 colors Bayla can use to make a bracelet.

Bayla made a *Simple Loop* bracelet for Griffin using 10 black bands and 11 red bands. For Myoko, Bayla made a more intricate *Starburst* bracelet using only red and black bands in the ratio 27:57. The fewest number of bands Bayla could have used to make Myoko's bracelet is how many times the total number of bands Bayla used to make Griffin's bracelet?

With a ratio of red bands to black bands of 27:57, the fewest possible number of bands needed to make Myoko's bracelet is 27 + 57 = 84 bands. We are told that Griffin's bracelet is made with 10 + 11 = 21 bands. Therefore, the fewest number of bands needed to make Myoko's bracelet 84/21 = 4 times the number of bands needed to make Griffin's bracelet.

Bayla's favorite bracelet design, the *Ladder*, requires a total of 76 bands. Bayla decides to use all of the bands she had previously set aside to make a *Ladder* bracelet to make a *Simple Loop* bracelet instead. She noted that a *Simple Loop* bracelet, made of 21 bands, measures 12.5 cm from end to end, when straightened, as shown. (Note: Bayla's measurement does not include the clasp.)



Based on this, when Bayla's *Simple Loop* bracelet, made of 76 bands, is straightened and measured from end to end, not including the clasp, what would be its length, in centimeters? Express your answer as a decimal to the nearest tenth.

One way to solve this problem is using proportions. If we let c represent the length of the Simple Loop bracelet made with 76 bands, we can write the following proportion: 12.5/21 = c/76. Cross-multiplying and solving for c, we get $21c = (12.5)(76) \rightarrow 21c = 950 \rightarrow c \approx 45.2$ cm as the length of the Simple Loop bracelet made of 76 bands.