

# Developing Maths Eyes A Resource Pack

An Innovative Approach to Building a Positive Image of Mathematics



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# "The real voyage of discovery consists not in seeking new landscapes but in having new eyes"

Marcel Proust French Novelist and Author, 1871-1922

#### **Foreword**

The underlying philosophy of Maths Eyes is that mathematics is everywhere in our real world. The omnipresence of mathematics provides glorious opportunities for human beings at all stages of their lives. The description of mathematics as a "most useful, fascinating and stimulating division of human knowledge" in the Primary School Curriculum, 1999 is most appropriate. Children and all learners have the capacity to luxuriate in the creative activity, in the investigative nature, in the sense of awe, wonder and excitement of mathematics when they are permitted to imbibe its essence and to experiment with its everyday possibilities. This conceptualisation of the everyday world of mathematics as something creative, interesting and satisfying does not end with the onset of adolescence, it has the capacity to inspire, sustain and assist us as teenagers, as adults and as senior citizen. Consequently Maths Eyes – A Resource Pack is of interest to a variety of audiences along the lifelong learning continuum.

Learners using Maths Eyes – an Innovative Approach to Building a Positive Image of Mathematics Handbook will enjoy their journey as they embark on maths trails, carry out maths investigations, make sense of maths problem pictures and posters and keep a maths diary. But they will be challenged to, figure out divine proportions and the golden ratio, walk mosaic floors and footpaths to crack complicated codes and to deciphering the language of the maths world.

Using a resource pack such as Maths Eyes will enable learners to engage in critical thinking, to develop problem solving skills and to become rigorous investigators. They don't need to search in books, libraries or cyberspace to integrate mathematics into their lives in a most enjoyable way, they need merely to sharpen their mathematical senses, to look around and to take the time to make the connections. In their own environments learners will enhance their mathematical literacy by, designing as artists, experimenting as scientists and exploring space and place as historians and geographers. As they experience the relevance, the interconnections and the integrative properties of all knowledge they will construct their own unique knowledge and will emerge as confident individuals capable of solving the problems of their own situations in creative and critical ways.

Dublin West Education Centre is pleased to collaborate with the Institute of Technology Tallaght in this innovative and stimulating initiative. Maths Eyes is the brainchild of Terry Maguire. It is the first of a series of interesting maths initiatives. I congratulate Terry and her team; Ciarán O'Sullivan, John O'Mahony, Mairéad Ryan, Marie Morgan and Chris Meehan. I thank Ciara O'Donnell for her wise and stimulating counsel.

I am honoured to commend Maths Eyes as a resource that will not only make a difference to maths education, but will also be a model for lifelong learning.

Gerard McHugh
Dublin West Education Centre

## **Resource Pack Development Team**

#### **Terry Maguire Institute of Technology Tallaght Dublin**

Dr Terry Maguire has been involved in education for almost 25 years. She is currently the Head of Lifelong Learning at the Institute of Technology, Dublin, Ireland. Her research interest includes identifying good models of professional development for teachers of mathematics, good practice in teaching and learning mathematics, and uncovering the hidden mathematics of the workplace.

#### Ciarán O Sullivan Institute of Technology Tallaght Dublin

Ciarán O'Sullivan is a Mathematics lecturer in the Dept of Mechanical Engineering, IT Tallaght. He has been mathematics educator for 27 years, the last 13 years at third level and the previous 14 years at second level. He is interested in engineering mathematics education, in particular how to engage learners with mathematics in real world contexts and how to use mathematics as a vehicle for civic engagement. Currently he is completing the customisation of a suite of 'Green Mathematics' lessons for use in primary and adult education.

#### **Chris Meehan**

Dr Chris Meehan has been involved in early childhood education for many years. Here it is well agreed that children learn best when learning from their own experience and environment. A long time dabbler in Mathematics he holds a Ph.D in Algebra.

#### John O'Mahony

John O'Mahony is a Support Teacher in Sacred Heart Senior School, Killinarden. Prior to taking up his present post of ICT Co-ordinator he held the post of Mathematics Co-ordinator in the school. He believes that for success with Mathematics in primary schools, children must see Mathematics not only as a school subject but as being relevant to their own everyday lives.

#### Mairéad Ryan

Mairéad Ryan is principal of Scoil Maelruain Junior, Old Bawn, Tallaght. She has completed research on the use of games to enhance mathematical thinking and understanding.

#### Marie Morgan

Marie Morgan is a Maths Recovery teacher in Holy Rosary P.S. in Ballycragh, Firhouse. In her school, she is involved in the teaching and promotion of maths in the early years. She is completing her Masters of Education on digital learning in St.Patrick's College,Drumcondra. She believes to be successful in teaching maths "we must always start at the child's world and bring it to the maths world".

#### Ciara O'Donnell

Ciara O Donnell works with the Professional Development Service for Teachers (PDST) which provides Continuing Professional Development for primary and post primary teachers. Ciara has worked as a co-ordinator with teacher support services since 2006 leading the delivery of in-service and school support following the introduction of the Primary School Curriculum. She was the Deputy Director with the former Primary Professional Development Service (PPDS) with responsibility for curriculum development. Ciara has a special interest in Mathematics, and studied it at third level as part of her Bachelor of Education degree. Ciara holds a Masters degree in School Leadership with particular focus on curriculum development and implementation.

#### The Resource Pack

This pack is a resource to help teachers or tutors to bring the "real world" into the classroom. It is not a series of lesson plans but rather a collection of ideas to adapt and build on for use with all age groups.

#### **Developing Maths Eyes Means:**

- New ways of looking at familiar things
- New ways of considering familiar things
- New insights into our daily lives, leisure and work
- New approaches to teaching and learning

#### **Structure of the Resource Pack**

The resource pack includes 6 Maths Eyes Posters as example of "Real World" starting points for introducing mathemathics.

#### Six Maths Problem Posters

- Gingerbread men
- Hopscotch
- Mosaic Floor
- Footpath
- Airport
- Butcher Shop window

#### **Guide to Developing Maths Eyes**

The guide to developing Maths Eyes, includes all the following resources:

- A resource website (www.haveyougotmathseyes.com) for supporting the development of 'Maths Eyes'
- Selection of Maths Eyes pictures linked to mathematical themes
- A selection of Maths problem pictures (with exhibition guide)
- 10 Maths Eyes 'Community Posters'
- · Solve it exhibition and interaction challenge

The pack can be used with adult learners and includes a number of activities for parents or guardians of children to work together to develop their Maths Eyes.

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# **Developing Maths Eyes - A Resource Pack**

#### Introduction

Everybody uses mathematics in their life but sometimes they don't call it mathematics. These 'everyday' mathematics skills often involve the use of complicated mathematical ideas and techniques. Individuals underestimate their mathematical abilities because they often consider the mathematics they can do as 'common sense' and the tasks they can't do as 'mathematics'.

Developing mathematical eyes allows individuals to see the different types of mathematics that they are doing everyday. Having "Maths Eyes" also helps people to understand that mathematics is more than the skills and formulae they might associate with their school experience. Furthermore, developing "Maths Eyes" encourages discussion about, and language development around, mathematics.

This pack includes many activities to help to develop "Maths Eyes". The range of activities include going 'out and about', talking about pictures and photographs; 'Learning with Maths Problem Pictures'; doing maths investigations; developing a Maths Trail and keeping a Maths Diary.

#### The Real World Maths World Cycle

Viewing the world through "Maths Eyes" provides excellent opportunities to help learners to develop a 'competence with the use of mathematical language, knowledge and skills'.

For most people the mathematics that surrounds them in their daily lives remains 'invisible'. When thinking about sport people often focus on 'number' or 'data handling' e.g. scoring systems, betting and records of sporting achievement over time. Other kinds mathematics remain hidden and perhaps not as immediately obvious e.g. symmetry, shape, pattern and relationships.

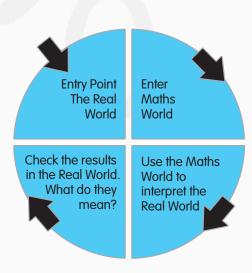


Figure 1. The Real World Maths World Cycle

Often in activities using mathematics, the starting point for an activity is the rule or formula that must be used to calculate the answer. Developing "Maths Eyes" provides opportunities to use the Real World as the starting point and context for further development (Figure 1). The Maths World is viewed as a tool to help understand and explore the Real World.

Take for example a football pitch as the starting point for a group activity. **The group could be asked to discuss:** 

- What do the markings on the surface mean?
- Why are they needed?
- Are they all needed, if so, by whom?
- Where are they located on the surface (21yard line, centre point)?
- How do these markings affect what happens on the pitch?
- What would happen if...?

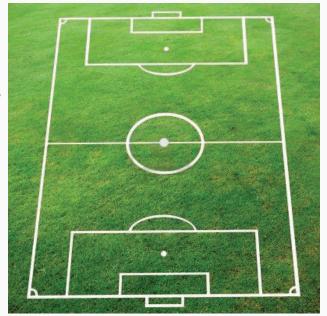


Figure 1

**Note:** Imperial measure is used for Soccer, GAA and Rugby use metric. There are many sports each with there own markings that could be used as starting points e.g. Basket Ball, Tennis etc.

The activity could include asking the group to measure the pitch. Initially learners might be encouraged to use their own strategies (e.g. non-standard units, strides, hands etc.) before units of measurements, metre sticks, or measuring tapes are introduced (e.g. learners might use strides). Estimation should be encouraged as it helps to build confidence and number sense. The Real World does not necessarily demand pinpoint accuracy, words like around, about, and approximately are all acceptable words most of the time. These are typical of the words we use for everyday estimates as part of our daily lives.

However, sometimes measurements have to be accurate. In the world of sport there are many examples of when accuracy is essential. In athletics or swimming accuracy to one hundredth of a second can separate winners from runners up.



Figure 2
Swimming pools, if they are to be used in a competition, have to be an exact length and width.
The following type of questions could be used to stimulate discussion about measurement;

- Why is this level of accuracy sometimes necessary?
- Does the local swimming pool meet these strict criteria?
- How do other sports measure up (think about the impact of non-standard Soccer or Rugby pitches)?

When learners are working in the Maths World they can be encouraged to use and become familiar with the language of the Maths World. However maths language should only be introduced in context when the task demands the use of standardised units or tools. Once learners have explored the concept in the Maths World they should be encouraged to use what they have learnt in the Maths World to make more sense of what is happening in the Real World. For example:

- Which swimming pools are competition standard pools?
- What does this mean about their length and width?

In this way the relevance of mathematics becomes more apparent, learners develop more understanding and the Maths World has real meaning.

# **Techniques for Developing Maths Eyes**

This guide to developing 'Maths Eyes' will focus on five different approaches:

#### Section

- 1. Maths Pictures or Posters
- 3. Maths Problem Picture Exhibition
- 5. Keeping a Maths Diary

#### Section

- 2. Maths Trails
- 4. Maths Investigations

#### **Introduction: Seeing more than Number**

When using the environment as a resource for developing Maths Eyes, individuals often focus first on quantity and number. Consequently as teachers or tutors help to develop "Maths Eyes", it is useful to think about what might be seen under the following mathematical themes.

Number (Including Algebra and Measures)	This is the first theme that people see when they look with mathematical eyes. This theme covers all the things that relate to number in everyday life e.g. prices, car registrations, numbers associated with buses, the telephone, lotto, barcodes etc.	
Space and Shape	With Maths Eyes we see the range of shapes that make up our everyday world e.g. rectangular windows, round wheels, square tiles, spherical balls, boxes (cuboids) pipes (cylinders), types of triangles, symmetry etc.	
Data handling and Chance	You only have to open a newspaper to see the range of data in charts and tables that we have to interpret everyday e.g. survey results, trends in the stock market or price of petrol, sports results, timetables or league tables.	
Patterns and Relationships	Pattern surrounds us in our daily lives e.g. road markings, window panes, railings, tiles, wall and floor coverings, ratio/fractions etc.	
Problem Solving	In our everyday lives we constantly solve a range of problems Problem solving is an important mathematical and everyday skill. The kinds of problems that can be included are e.g.  • how tall is the tree?  • how to work out how fast the traffic or a river is flowing  • can you workout a family tree from the information listed on a gravestone?	

# **Section 1. Maths Problem Pictures or Posters**

#### Introduction

Using maths problem pictures or posters is an excellent way to help learners to develop their "Maths Eyes". Maths pictures are snapshots of familiar things that capture some aspect of mathematics.

The best pictures are those that the group can relate to and which stimulate interest:

- Something the group can relate to e.g. pictures of their own area, of things they are familiar with such as, doughnuts, chocolate bars, prominent railings, pathways, buildings or statues.
- Pictures that are selected to meet the knowledge and skills of each group are especially useful in a differentiated classroom.

#### **Getting Started**

Maths Problem pictures provide an excellent opportunity to get learners to see the world around them through new "Maths Eyes". The process should not be rushed nor should the mathematics be introduced too quickly. Give the learner time to develop their eyes rather than see the picture through your eyes.

The pictures are intended to stimulate discussion on topics such as Pattern, Interpretation, Decision, Shape, Measurement, and Number etc. The initial 'prompt' questions should be very general and appropriate to any category of photograph. Follow-up questions can be used to guide the class/group in focusing on a particular aspect of the image. Groups need to be given time to discuss the picture or poster amongst themselves first. They should be asked to record their responses and then feed back to the class. Learners should be given a maximum of 2/3 questions depending on their group.

#### **Prompt Questions - Examples**

- What do you see in this picture?
- Where do you think this picture could have been taken?
- Can you talk about what is happening in the picture?
- Can you think of story to tell about the picture?
- Look at the picture with your "Maths Eyes", what can you see?
- Can you use any maths words to describe something in the picture?
- Can you think up any maths questions for other groups?

# Poster 1 Gingerbread Men

(Included in the Pack)

#### **Sample Questions**

Remember to start with general questions about the picture:

- What do you see in this picture?
- Does anyone know any story they could tell about a gingerbread man?



- What do you need to make gingerbread men?
- Who thinks gingerbread men are good to eat?
- Which ingredients are good/not good for people to eat?
- Which part of the gingerbread man would you eat first?
- What are the gingerbread man's eyes made of?
- What are his feet made of?

Other questions

- Are there other things you could have made the feet with?
- How many legs/eyes/arms/buttons does each gingerbread man have?
- How many gingerbread men are in the first row?
- How many gingerbread men in the poster altogether? How did you work this out?
- What colour are the eyes of the gingerbread men?
- What colour buttons can you see?
- How many smarties were needed to give the two buttons to each gingerbread man?
- Can you think of questions about the picture to ask other groups (question starters could include: How many..., how many more...which colour etc.)?

Other questions.			

# Poster 2 Hopscotch

(Included in the Pack)

#### **Sample Questions**

Remember to start with general questions about the picture

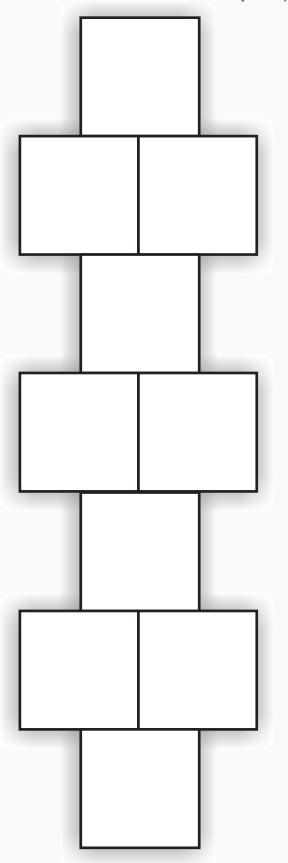
- What do you see in this picture?
- Does anyone know any story they could tell about what they see?
- How many different colours can you see?
- What colours do you see?
- What does this colour combination remind you of e.g. County colour?
- What shapes do you see?
- What is that shape called?
- How many blue blocks?
- How many red blocks?
- How many yellow blocks?
- What shape do the yellow and blue blocks joined together make?
- If you added more squares, what would you add to keep the pattern?
- Where would you put any new squares you might add to the picture?
- What game could you play using these shapes?
- What does this picture remind you of?
- How would the picture have to change to play hopscotch?
- What way would you number the squares to play hopscotch?
- How do you play this game?
- How many hops do you need to get to the top?
- If I was on the first blue square how many steps would I take to get to......?
- If you hop up to the top and back again how many hops altogether?
- Can you find a line of symmetry in this picture?
- Trace the steps you would take on your own hopscotch diagram (template attached).
- Why do the shapes at the top of the picture appear narrower than the bottom?
- Can you think up a maths problem based on this picture for the other groups?

Other questions:			



# **Template**

(This can be drawn outdoors if required)



# Poster 3 Mosaic Floor (Included in the Pack)



#### **Sample Questions**

Remember to start with general questions about the picture

- What does this picture remind you of?
- Can anyone make up a story about what this picture shows?
- Where might you find this kind of pattern?
- Put on "Maths Eyes", what do you see now?
- What shapes can you see?
- What is the smallest shape in the picture?
- Can you find a shape that is just one colour only?
- Can you see shapes with more than 2 colours?
- What squares can you see made up of different colours?
- Can you see a line of symmetry for one of the squares?
- Where else might you find this shape?
- Choose a square and describe the pattern.
- What pattern is in the squares?
- What size angles can you see?
- What kinds of angles can you see?
- Can you use maths words to describe what you see? (Teacher or tutor could record all the maths worlds words used)

•

#### **Suggested Related activities**

- Replicate the pattern using different colours.
- Describe the pattern in the white and red square.
- What fraction of the tile was used to make a triangle?
- How many different size white triangles can you see?
- Give a group a logic block to make the pattern.
- Introduce the term Tessellation.
- Investigate what shapes match together.
- Could a circle shape fit in the pattern?

Other questions:			

# **Poster 4 Footpath**

(Included in the Pack)



#### **Sample Questions**

Remember to start with general questions about the picture.

- What does this picture remind you of?
- Does anyone know what the yellow squares might be used for on a footpath?
- Describe what you see.
- What would the different surfaces feel like to walk on?
- What shapes can you see?
- How could you count the circles on each square?
- Is there a quick way of counting them?
- Is there the same number of circles on each square?
- How many circles are hidden in the bottom yellow squares?
- What other shapes can you see (irregular pentagons, triangles, hexagons)?
- Can you see any straight lines on any shapes?
- Can you see any square numbers?
- Can you see any other square numbers?
- Can you use maths words to describe what you see?
- Is the pattern on all the yellow squares the same?

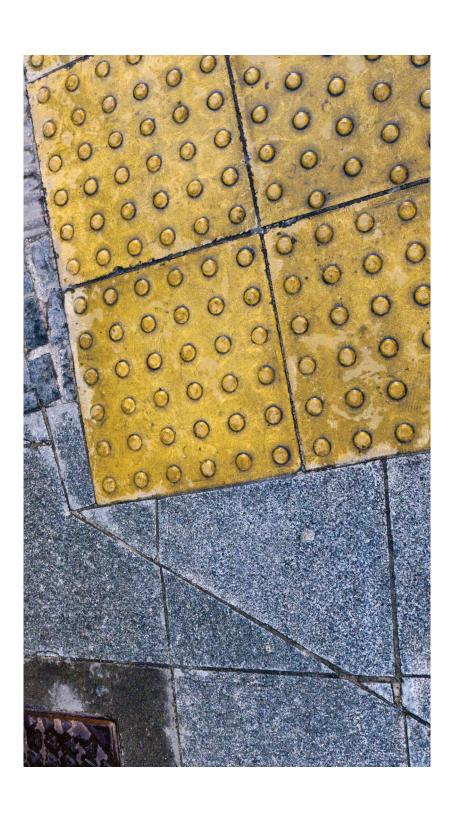
#### **Suggested Related activities**

- Ask learners to look around when they are out and about to see what other patterns there are on yellow paving stones.
- Use the square block, identify square numbers or play a game e.g. join the dots (like the pattern on yellow square. Template attached); Develop your own pattern.
- Extend the pattern in the picture by completing the pattern in the clear area in the template attached.

# **Template**

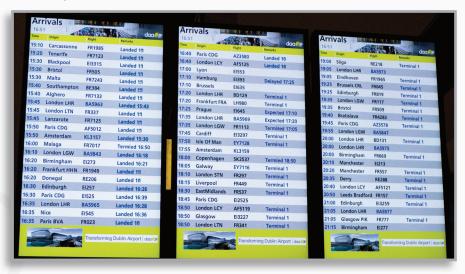
Complete the pattern in the white area.

# **Template**



# **Poster 5 Airport Departure Timetable**

(Included in the Pack)



#### **Sample Questions**

Remember to start with general questions about the picture?

- What do you see in this picture?
- Where might you see this kind of picture?
- What information does it give?
- How is the information on this picture useful?
- What do you think the codes mean?
- How do you know that a plane is from a particular airline?
- How many different airlines have planes arriving at this airport?
- Where have the planes come from (cities, countries)?
- Using a map of the World, mark in all the cities from which planes have arrived.
- Which country has the most/least planes arriving?

#### Suggested Related activities:

- Work out how many planes land in an hour are there busier/quieter times?
- Workout which airline has the most/least planes arriving/departing.
- Draw a bar chart of the number of planes from each country/city.
- Use sets to group counties/cities/continents together.
- List the planes that landed on time/early or delayed.
- What was the time difference between here and the places that the planes arrive from?
- What time is the earliest/latest plane arrive/due from e.g. London, Glasgow?
- Use the internet to record arrivals/departures from your main airport.
- Can you spot any spelling mistakes in the picture?

# **Poster 6 Butcher Shop Window**

(Included in the Pack)



#### **Sample Questions**

Remember to start with general questions about the picture.

- What do you see in this picture?
- Can your group make up a story/poem/draw a picture about this kind of shop?
- What would you buy in this shop?
- What is the name of the job a person does in a shop like this?
- When is this type of food usually eaten?
- Do you like to eat in this shop?
- What different animals does the meat come from?
- How many people are in the shop?
- What are the people doing?
- What kind of meat do you think the women might be buying?
- Why is meat kept in a refrigerator?
- What is behind the counter?
- Why do you think this kind of shop used has a mirror like this?
- What shape is the mirror?
- What time of year do you think it is?
- How did you work this out?
- What is the difference between the highest and the lowest priced offer?
- What could you buy with €10.00 in this shop?
- Why would the butcher say €9.99 rather than €10.00?
- Pick 3 things the women might buy, how much would it cost altogether?
- Now try and buy three things as cheaply as possible.
- How much would you save by buying 10 chicken fillets?

# Section 2. Maths Trails

#### **Introduction – New Eyes**

Once learners have started to use their "Maths Eyes", Maths Trails are a great way to help learners to develop their eyes further. The steps in developing a Maths Trail are outlined below.

#### Steps in constructing a maths trail

- Walk the trail with a group and encourage them to share what they can see through their "Maths Eyes". Take notes as you go along include ideas for activities. This can be describing a shape; calculating the number of railings in a fence; working out the height of a tree or the rate of flow in a stream or playing hopscotch.
- Write out the notes you took in more detail identifying key stops you want to include.
- Take a photograph of something recognisable at each stop and decide what you want the individual taking the trail to do at this stop.
- 4 Include tasks at each stop for different levels of ability to keep everyone engaged and interested.
- Give each stops a name e.g. the triangle stop and focus on this particular aspect in the information/tasks that you include in that stop on the trail.
- Intersperse the maths discussion with real life discussion. For example, you might want to point out symmetry or lack of symmetry in the different leaves of different types of trees. Identify and name the trees and their characteristics.

## Things to consider then constructing a trail

- 1. Try and make the trail a circuit that starts and finishes at the same place.
- 2. Choose the direction you walk with care e.g. avoid the temptation of passing a playground early in the walk!
- 3. Give learners enough to do to keep them interested and engaged.
- 4. A good trail lasts about 45 mins to 1 hour.
- 5. Get individuals to work in groups of 3 or 4 to help figure out some of the tasks.
- 6. Make sure everyone has fun.
- 7. Make the trail available on line for others to use in their own lives with family and friends.
- 8. Encourage participants to bring a camera to start their own Maths Eyes collection.
- 9. Cover a variety of maths themes.
- 10. Ensure each group has the equipment needed for an activity.

Maths Trails are ideal opportunities for learners to use and develop their own strategies for solving problems. The following activities give some examples of how learners might be encouraged to develop their problem solving strategies before using standard measurements.

#### **Examples of trail activities that could be incorporated**

- Identifying lines of symmetry in various leaves
- Investigating how fast a stream is flowing
- Working out a strategy for estimating for example the number of birds on a pond
- Developing a strategy for measuring distance e.g. kid steps, or paces
- Working out the height of a tree or statue by comparison or estimation
- Once learners are accustomed to trails they should be encouraged to write their own.

#### **Link to Maths Trails on the Web**

www.irishtrails.ie

www.ncetm.org.uk/search?q=maths+trails

www.ncetm.org.uk/public/files/262659/Function\_Art.doc

www.trails.ie

www.mathsweek.ie

Maths Trail have been developed for Seán Walsh Park and Corcagh Park in Dublin 24, and can be downloaded from http://www.southdublinlibraries.ie/childrens-zone/

## Section 3. Maths 'Solve It' Exhibition

This pack provides you with a sample 'Maths 'Solve It'. The exhibition consists of 19 maths problem pictures and 10 posters. Thumbnails of a range of the posters are shown below:

































In addition, an interactive challenge for learners for this 'Solve It exhibition' has also been developed and is available to download from **www.haveyougotmathseyes.com**. This challenges and encourages learners to look at the exhibition with particular questions in mind.

#### For example.

- 1. Which work has the most colour in your opinion?
- 2. Find a work you think has a peaceful atmosphere.
- 3. Which is your favourite poster and why?

Learners should be encouraged to take their own photographs and develop 'maths problem pictures' for their own exhibition.

To help those visiting the exhibition to interact with the exhibition, it would be helpful to develop an 'interactive challenge' for your exhibition.

# **Section 4. Maths Investigations**

Maths investigations are a great way of opening "Maths Eyes". The key is to let participants use their own strategies for doing the investigation.

Three selections of maths investigations are included:

- Building Number Sense
- Geometry Sense of Proportion
- Problem Solving and Stories

For each of the three types of maths investigations, a simple investigation along with more detailed investigations are included.

These investigations should be adapted for use with different groups to suit their particular level of ability.

#### **Developing Number Sense:**

Simple Investigation Associated Investigations - Just a minute

- Mind over Matter

- Mix and Match

- Decimal Point – Just a point?

- Speedometer

### **Geometry - Sense of Proportion:**

Simple Investigation
Associated Investigations

- Card Challenge

- Leonardo's Ratios

Divine ProportionsThe Golden Ratio

#### **Problem Solving and Stories:**

Simple Investigation

- Great wages

**Associated Investigations** 

News or sportThe Bath story

- Who lives where?

- Crack the Code!

# **Developing Number Sense**

#### **Teacher and Tutor Guidelines**

Number sense is an intuitive feel for numbers and their relationships. Think, for example, about the size of a gram compared to a kilogram, the number of 20 cent coins to make a euro, or how big a billion is compared to a million.

Number sense is an essential part of everyone's daily mathematical life and it grows slowly and develops over time. The ability of an individual to estimate depends on their number sense. To be good at estimating an individual must have a good number sense and an understanding of the relative difference between different quantities e.g. grams and kilograms; metres and kilometres; millilitres and litres.

Estimation is a key skill in mathematics and for everyday life. When we estimate we find an approximate answer, rather than an exact answer. To be able to estimate we need two different skills. First, we use techniques like rounding e.g.

Exact - Mary's new jumper cost €49.95Estimate - Mary's new jumper cost almost €50

The harder skill is distinguishing how accurate one needs to be in any given situation. For example if you were buying a new window, it is important that you have an accurate rather than estimated measurement of the size of window you need.

When individuals estimate they use a range of words that let other people know that we are not being exact. These words include: nearly, almost, about, just over, just under, approximately, a little bit less than, a little bit more than. These words should be displayed in the learning environment and learners should be encouraged to use them regularly.

# **Developing Number Sense - Simple Investigation Just a Minute**

#### **Teacher and Tutor Guidelines**

A selection of learners are chosen and asked to raise their hand and close their eyes. They are requested to put their hand down after they think one minute has passed.



#### OR

Teacher or Tutor stands up with learners watching then and sits down after a period of time (e.g. 37 seconds). Learners are invited to estimate how many seconds the Teacher or Tutor has been standing. Estimates could be collected together to consider the range, mean median etc.

#### **Equipment**

Stop watch or mobile phone.

## **Associated investigations**

Number Sense 1 Mind over Matter

Number Sense 2 Mix and Match

**Number Sense 3** Decimal Point – Just a Point?

Number Sense 4 Speedometer

# **Developing Number Sense 1 - Mind Over Matter**

#### **Teacher and Tutor Guidelines**

A good strategy to employ with this investigation would be a pair/share strategy and followed by snowballing the answers in the whole class.

**For pair/ share**, the students are grouped by the Instructor into twos or threes and then given time to do the activity of matching the cards or filling in the question sheets as follows:

- 1. Whole class listens to the class instruction regarding the task.
- 2. Learner pairs then get 2 minutes to think about the activity.
- 3. This is followed by 5-10 minutes where the pairs discuss and complete the task as best they can together.

#### **Snowballing the answers:**

At the end of the ten minutes the Teacher/Tutor then assembles the overall picture on the board by getting each pair to volunteer one answer. Before the answer is written down, other pairs get a chance to discuss so that everyone is satisfed that the suggested answer is reasonable.

#### **Additional Activities for all Number Sense Activities**

- What did you need to use from what you learnt before to make your estimate?
- What strategies did you use to estimate?
- How could you check your answers?

Adapt items on the templates provided to suit your learners or design your own templates.

#### Some answers: Mind over Matter Worksheet

The size of an egg: Small about 50g, medium about 57g, large about 65g

The distance from Donegal to Killarney by road is about 406 kilometres (according to AA route planner).

A bucket holds about 10 litres.

# **Developing Number Sense 1 - Mind over Matter Worksheet**

Working in your pairs or small groups try to produce reasonable estimates to the questions below using only metric units. Write down the main discussions/issues raised your team had about each challenge.

What is the weight of an eg	q?
	3-
What is the temperature of	the room today?
How far is it from Donegal t	to Killarney? (use atlas/map if required
How long is a match?	
Leve binds in the collings	$\sim$
How high is the ceiling?	
What is the weight of a buc	ket of water?
fovervene in this reem lev	head to toe, how far would we stretch

(Based on approach from Marr, B. and S. Helme 1991. Breaking the Maths Barrier, Department of Employment Education and Training: Canberra.)

# **Developing Number Sense 2 - Mix and Match:**

#### **Teacher and Tutor Guidelines**

Print the Mix and Match template on card.

Cut out the clues and answers and ask the learners to match each clue with the most suitable answer. Use the pair/share strategy and then snowballing the answers as described in Number Sense 1 Mind over Matter.

Remember to ask learners to make sure they are able to give a good reason for the answer they have chosen.

#### Notes:

Adapt clues to suit your learners.

Chair weight range is approximate but can be used to promote discussion. Investigate strategies to find the weight of a chair. To estimate the weight of a chair get a volunteer to weight them selves then hold the chair and take their weight again. Work out the increase in weigh, this is weight of chair.

The capacity of a school bag is based on bag 30cm wide X 35cm high X 20cm deep with books.  $30 \times 35 \times 20 \text{ cc} = 14000\text{ml} = 14\text{L}$ 

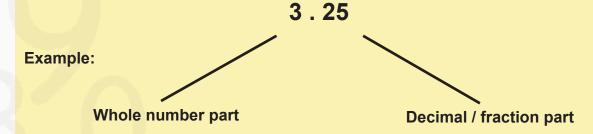
# Developing Number Sense 2 - Mix and Match Template

	The height of a doorway	2.5 metres	The width of a hand	8 centimetres
	The weight of a chair	Between 3 – 7 kg	The weight of one litre of water	1 kilogramme
	The width of a little finger	1 centimetre	The weight of a medium egg	57 grams
	The length of a swimming pool	25 metres	The capacity of a school bag	14 litres
	The weight of a 1 euro coin	7.5 grammes	The volume of a mug for coffee or tea	300 millilitres
		The width of a door	1 metre	

# Developing Number Sense 3 - Decimal Point: Just a Point?

#### **Teacher and Tutor Guidelines**

Explain to the group that the decimal point is used to separate the whole number part from the fraction part or decimal part of that number.



In Ireland and the United Kingdom the decimal point is written as a dot e.g. 3.25. In Europe the decimal point is written as a comma e.g. 3,25.

Sometimes things just don't make sense if there are no decimal points.

Give the group an example and get them to discuss whether it is true or not.

#### 'John was 16 metres tall'

- Is this a reasonable height for John?What is missing?
- Where should it go?

Re-write the statement correctly.

#### 'John was 1.6 metres tall'

Then read the story "Peggy Goes Shopping" (this story is provided as an example). Teachers or tutors should adapt/write a 'story' to suit their learners. Simple sentences can be used instead of a story to reduce the literacy requirement for learners as appropriate.

#### When you have finished reading the story:

- Organise the learners in groups of two or three and ask them to read the story again and put in the decimal point where they think it is needed to make the story make sense.
- Ask the learners to write another paragraph to the story about what Peggy did for the rest of the day. Try to include some numbers with decimal points.

- Give each learner a piece of paper. Ask the students to write three sentences based on their own life which need the reader to put a decimal point in the right place. Gather the sentences, copy them and then get the class in their groups to put the decimal points into the sentences.
- 4 Younger learners could be asked to remove a digit to make a story make sense.

# Sample Story

# **Peggy Goes Shopping**

Peggy decided to go out shopping for dinner. First she went into the butcher shop and bought 15kg of steak. She went into the vegetable shop and bought 25kg of potatoes and 1kg of carrots. The potatoes and carrots cost her €799. On her way home Mary met her friend Peter whose wife had just had a beautiful 375kg baby girl. Peter had visited them in the hospital 175 km away. It had only taken him 25 minutes to drive home as the traffic was very light. Peggy said goodbye to Peter and walked home.

#### Task:

Use decimal points to make sense of the story.

# **Developing Number Sense 4 - Speedometer**

#### **Teacher and Tutor Guidelines**

Give each learner a copy of the speedometer photograph (attached) and discuss with them what the photograph is of and where they find these dials. Discuss what the dials are used for and why they are important.

#### **Activity**

Give learners a copy of the speedometer template, and ask them to fill in the missing numbers. (Template attached)

Real life stimuli like the speedometer can be used to help learners make meaning of what they do in the maths classroom, e.g. what would this speedometer look like on a number line?



# Developing Number Sense 4 - Speedometer Template



### **Geometry - Sense of Proportion**

### Simple Investigation: Card Challenge

Cards numbered 0 - 9 are put into a hat or similar. Six digits are called out in turn. As a digit is picked out of the hat, each learner decides into which of the squares they will place that digit. When the six cards have been placed, the three two-digit numbers are added together. Those nearest to the answer set by the teacher or tutor, wins. Younger learners could be asked to make the biggest number. (Template attached)

### **Materials**

- Set of Big Cards numbered 0 to 9 (Template attached)
- Hat/bag

### **Associated Investigations**

**Geometry Sense of Proportion 1** Leonardo's Ratios

**Geometry Sense of Proportion 2** Divine Proportions

**Geometry Sense of Proportion 3** The Golden Ratio

**Geometry - Sense of Proportion -**Worksheet - Nearest Number

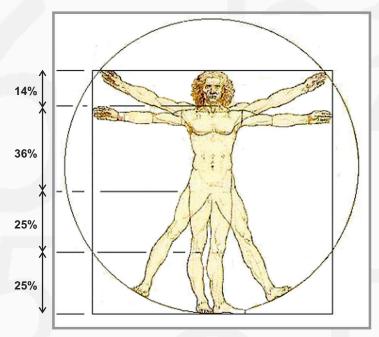
**Geometry - Sense of Proportion - Worksheet - Make the Biggest Number** 

### **Template**

### **Geometry - Sense of Proportion 1 Leonardo's Ratios**

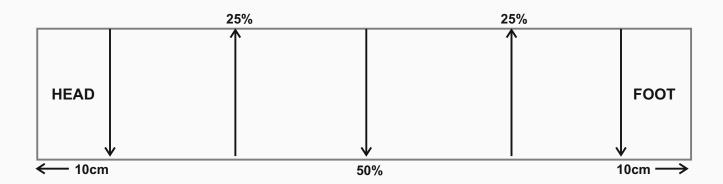
Leonardo da Vinci (1451-1519) was a great inventor, scientist and artist. Leonardo used science to improve his artwork. He studied to determine the "ideal" human figure. What he found was that we are not all that different after all! In this activity you can find out how you and your group fits Leonardo da Vinci's picture below.

The total length of your body (100%) is made up in the following proportion.



For this activity you will need wide elastic about 1m long.

Get together in groups of two or three. On the elastic tape mark a line about 10 cm from both ends. Mark one end 'head' and one end 'foot'. Fold the elastic over and mark the middle point as 50%. Fold the head line to meet 50% and mark in 25% on the fold. Similarly fold the footline end to meet 50% and mark 25% on the fold. Your elastic should be marked as illustrated:



### **Geometry - Sense of Proportion 1 Worksheet - Leonardo's Ratios**

Ask for a volunteer to be measured. Put your foot on the end of the elastic, and stretch it in a way which makes the 'head' mark reach the top of the head of the person being measured. Measure the different areas of the body described in the table below and estimate the percentage of the body in each area. Repeat the process for the other members of your group. Record your findings in the table below.

Measure	Learner 1	Learner 2	Learner 3
Top of head to neck			
Neck to top of leg			
Top of leg to under knee			
Knee to foot			

- Q1 Did your results agree with Leonardo da Vinci's findings?
- **Q2** Did the height of each person make a difference to the result?
- Q3 Can you think of any time in a person's life when their body might not fit Leonardo da Vinci's findings?

### **Geometry - Sense of Proportion 2 Divine Proportions**

### Face activity - Teacher and Tutor Guidelines

Divine proportions will help learners to recognise the proportions of the figure, face etc.

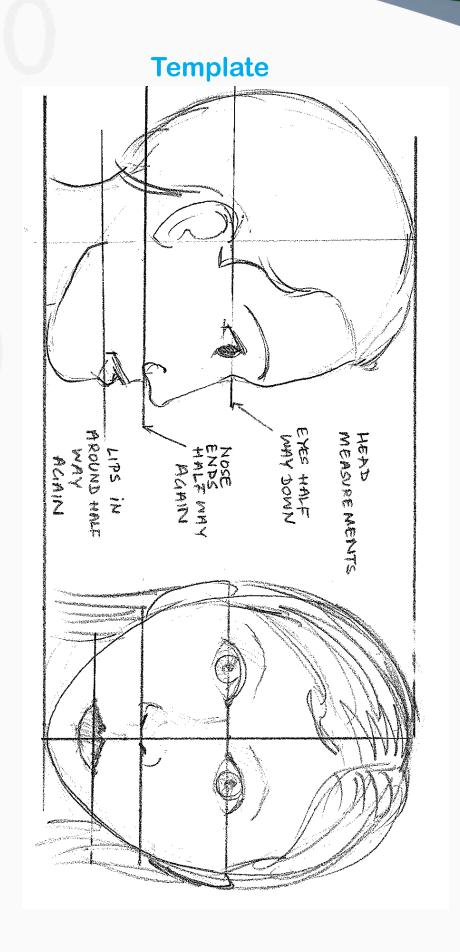
- First ask learners to look at the person beside them. Then ask where do they think their eyes are in relation to the head.
  - e.g. is it 1/4 way down, 1/3 way down?
- Ask one student to stand in the group.
- Fold a piece of paper –ordinary A4 will do- on the length so that it measures the same
  as their length from top of head to chin, it is easier to do this exercise from the side of
  the head.
- Cut the extra piece of off.
- Now hold it against the head again and put a light pencil mark on the paper to match where the eye is.
- Remove the paper and fold it in half on the length, unfold it and show the learners that
  the mark for the eye is half way down. So the eyes are at half way between the top of
  the head and chin.
- If you fold the paper in half again and open it, the new fold line will correspond with the end of the nose, and a further fold in half creates a line that lines up with the lips.
- Other measurements can be explored too. The ear starts on a line with the top of the nose and finishes on a line with the bottom of the nose.
- There is an eyes-width between your two eyes... i.e the distance between your eyes is the same as the width of your own eye.
- The full figure when standing is 7½ heads high, including the head. This one varies as a child grows to adulthood when standing.

### NOTE:

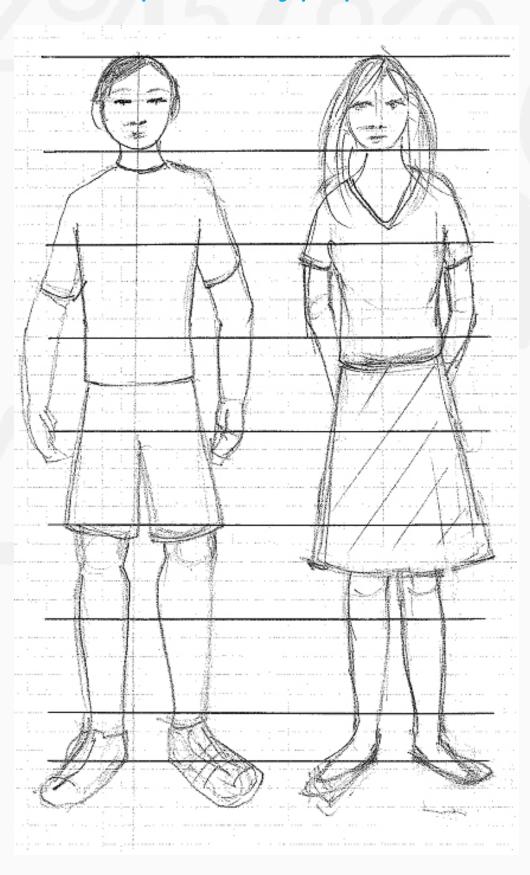
A person is  $7\frac{1}{2}$  times the height of their head (approximately) with proportions as shown in template of body proportions included.

### **Useful Links**

http://drawunglab.evansville.edu/body.html



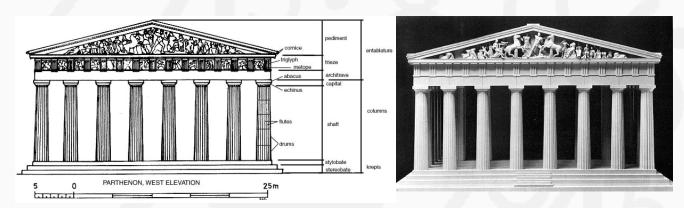
### **Template - Body proportions:**



### Geometry Sense of Proportion 2 Worksheet - Investigate these statements about the human body

1.	The average adult human is about 7.5 heads tall
	Answer: How did you investigate this?
2.	When you spread your arms along your shoulders (measure from the tips of your middle fingers), you measure about the same as your body height  Answer:
	How did you investigate this?
3.	The length of your foot is about equal to the length of your forearm  (The forearm is the part of the arm from the elbow to the wrist.)
	Answer:  How did you investigate this?
4.	The length of your face is about equal to the length of your hand.  Answer:
	How did you investigate this?
5.	Your eyes are separated by a distance of one eye width.  Answer:
	How did you investigate this?
6.	The distance from the bottom of your nose to the corner of your eye is equal to the height of your ear.  Answer:
	How did you investigate this?

### **Geometry - Sense of Proportion 3 The Golden Ratio**



The Golden Ratio is a special ratio that is approximately equal to 1.618:1. It has been used in art and architecture for centuries and is also found in nature. The Golden Ratio divides a line at a point so that the ratio of the smaller part of the line to the bigger part is the same as the ratio of the bigger part to the whole line. Also known as the Golden Mean this proportion is said to be the most pleasing to the eye. The Greeks were intrigued by this special mathematical relationship. Before them the Egyptians also had a sacred ratio; on the Great Pyramid of Gizeh in Egypt, the ratio of the height to half of a base is equal to 1.618:1. (Length; Height)

The Parthenon, part of the Acropolis in Athens, Greece, is one of the most splendid classical buildings, and is considered by many to be almost perfect. It was designed by the ancient Greeks as a shelter for the Goddess Athena. It has lost its entablature (the triangular bit at the top), but if that was included the building does conform to the Golden Ratio. If it does then this would show that as long ago as the fifth century BC architects were aware of the aesthetic qualities of the Golden Ratio. We do know that Renaissance artists used this special ratio calling it the Divine Proportion.

\_\_\_\_\_\_

Note: The ratio 1.618:1 is derived from Pythagoras's Theorem. Investigate this further using the internet, sources are given at the end of this section

In more modern times there are many examples to be found. The United Nations building, built in New York in 1952 is a modern example of the Golden Ratio being applied in architecture. It was designed by Le Corbusier. The ratio of the height of the building to the length of its base is 1.618:1. It contains 3 rectangles which are also in the Golden Ratio.



### **Other Sources:**

http://www.math.ubc.ca/~hoek/Teaching/Golden/Divina.html (good on spirals)

http://goldennumber.net/goldsect.htm

http://plus.maths.org/issue22/features/golden/

http://www.mlahanas.de/Greeks/GoldenSection.htm

http://www.goldenratio.org/info/

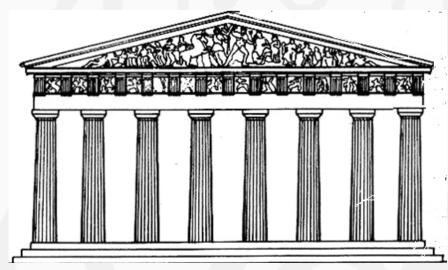
The giza pyramids:

http://www.pbs.org/wgbh/nova/pyramid/geometry/

### **Footnote**

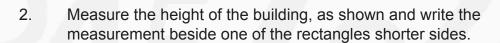
Based on ideas from :**How Mathematics Works (Dorling Kindersley Publishers Ltd)** by Carol Vorderman **ISBN:** 0751311596 **ISBN-13**: 9780751311594

### **Geometry - Sense of Proportion 3 Worksheet - The Golden Ratio**

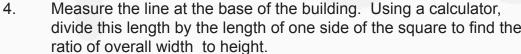


To carry out this investigation the following tools are needed; ruler, calculator, pencils and set square.

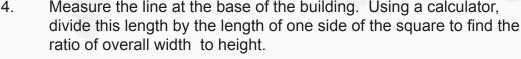
1. On a photocopy of the scale picture of the Pantheon, rule a rectangle around the outer limits of the building, including the triangle at the top, as shown on the right.

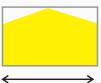


Let the height measurement equal one side of a square. Mark the height measurement on the rectangles longer sides, then draw a line on the shape to complete the square. Use the set square to check that this is a true square.









Answer: \_

5. At the bottom of the building measure the width of the rectangle marked in red. Using a calculator, divide the length of one side of the square by this new number, to find the ratio of the height of the smaller rectangle to its width.



Answer:

What do you notice about the two answers to question 4 and question 5?

### **Problem Solving and Stories:**

### **Simple Investigation - Great Wages**

A very wealthy business person comes up to you and says he will give you a €1,000,000 if you come and work for her for 3 weeks (21 days). Will you take the job?

On the first morning of the job she says that he is going to give you a choice about how you can get paid. Either she will give you €1,000,000 now or else she will pay you a euro a day doubling your pay each day for the 3 weeks.

Which would you choose?	
Give a reason for your answer.	

### **Problem Solving And Stories - Great Wages Solution**

Day 1	1
Day 2	€2.00
Day 3	€4.00
Day 4	€8.00
Day 5	€16.00
Day 6	€32.00
Day 7	€64.00
Day 8	€128.00
Day 9	€256.00
Day 10	€512.00
Day 11	€1,024.00
Day 12	€2,048.00
Day 13	€4,096.00
Day 14	€8,192.00
Day 15	€16,384.00
Day 16	€32,768.00
Day 17	€65,536.00
Day 18	€131,072.00
Day 19	€262,144.00
Day 20	€524,288.00
Day 21	€1,048,576.00

**Teacher and Tutor note:** A full ream of paper is a good starting point for this activity. How many euro notes would you make from an A4 sheet? How many sheets in a ream of paper? How much does a ream of paper weigh? How many reams of paper could a person carry?

### **Problem Solving and Stories: Investigating News or Spoof**

Larry arrives in on Monday morning and for 'news' time in class he volunteers the following story he heard from a friend:

"On Friday my friend saw a bank robber walk out of a bank with €1 million in small denomination euro notes."

Is this story likely to be true?
Give your mathematical reason for your answer.
Additional Questions:
What assumption did you have to make e.g did you assume €5/€10 notes?

**Teacher and Tutor note:** A full ream of paper is a good starting point for this activity. How many euro notes would you make from an A4 sheet? How many sheets in a ream of paper? How much does a ream of paper weigh? How many reams of paper could a person carry?

### **Associated investigations:**

**Problem Solving and Stories 1The Bath Story** 

**Problem Solving and Stories 2** Who Lives Where?

Problem Solving and Stories 3 Crack the Code!

### **Problem Solving and Stories 1 - The Bath Story**

### **Teacher and Tutor Guidelines**

This activity uses the story of somebody's bath to introduce the graphs of the maths world.

View the Bathstory on www.haveyougotmathseyes.com

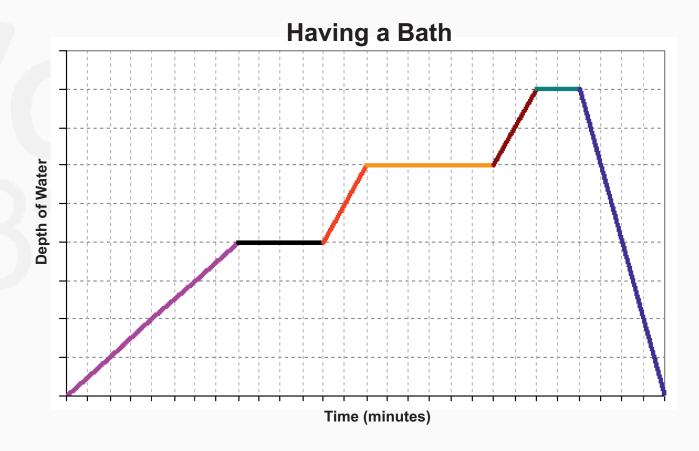
Ask somebody to volunteer or tell your own bath story to build a picture of what happens to the water levels when you have a bath. Focus on the water rising as the taps are filling the bath, then no change in the water level when the taps are turned off. When you get into the bath and the water rises again. Then no change until you add more water or get out of the bath or pull out the plug.

Go over the picture again explaining that in the real world the water is rising, as the taps are on and in the picture this is represented by the line going up. When there is no change the line goes straight across and when the water goes out the line goes down.

Ask teams to look at Patrick's Bath Story (worksheet 1) and write in their own "Real World" words the story of Patrick's Bath. Try not to use maths words in guiding them to write the story.

Once the students have talked about the picture of Patrick's Bath in their own words, introduce the learners to the "Maths World" using the guidelines provided later in this section.

### **Problem Solving and Stories 1 - The Bath Story Worksheet 1**

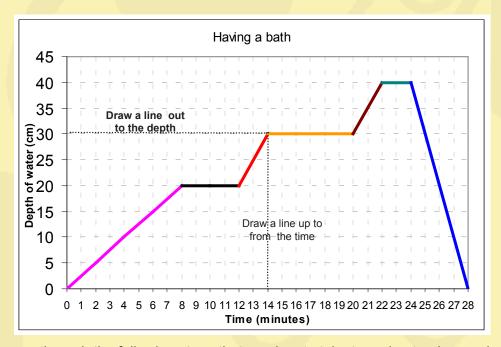


The Story	The Story of Patrick's Bath (in your own words):					

### **Problem Solving and Stories 1- My Bath Story Introducing the Maths World**

### **Teacher and Tutor Guidelines**

Introduce the Maths World and explain to the learners that in the modern world graphs and charts are a good way to communicate lots of information. Many people have difficulty interpreting graphs because they do not have the mathematical language to describe what is happening. Explain to them that in the "maths world" Patrick's Bath Story is called a line graph. Have the learners look at the following line graph (available as a template).

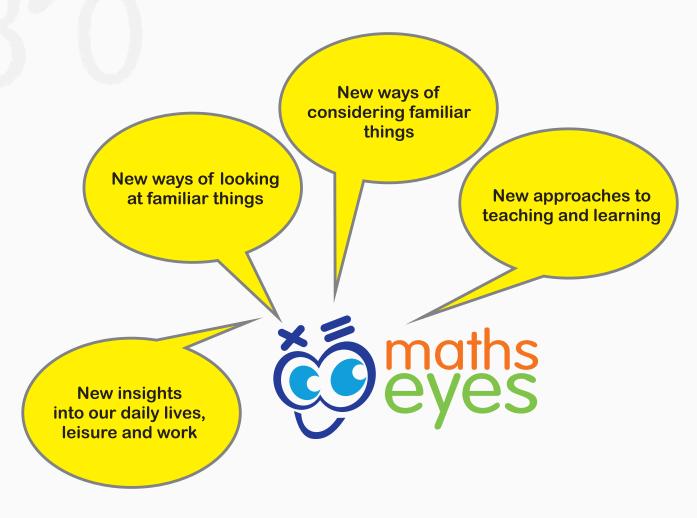


Take the learners through the following steps that one has to take to understand a graph:

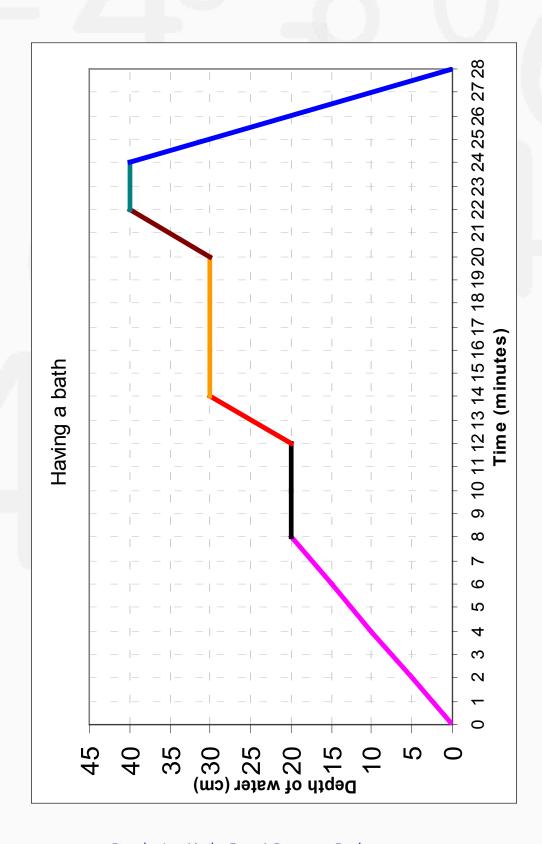
- **Step 1 -** Read the title of the graph (In this case the title is 'Having a Bath'). The title tells you what the graph is about.
- **Step 2 -** Read the label and the range of numbers along the side (called the scale or vertical axis in the Maths World). In this graph the numbers on the side tell you the depth of the water in the bath in centimetres.
- **Step 3 -** Read the label and the range of numbers along the bottom (called the horizontal axis in the Maths World). In this graph the numbers on the bottom tell you how much time in minutes have passed since the tap where first turned on.
- **Step 4 -** To work out the depth of water at any time draw a line from the time up to the line graph and then draw a line from the graph back to the side of the graph and read off the depth. The depth of water at 14 minutes is 30cm (look at the dotted line of the graph).

### **Introducing "Maths World" Language**

In the real world the words we might use to describe the pink line on the graph might include: 'getting higher', 'going up slowly', and 'getting deeper'. In the Maths World we would describe the pink line as **gradually increasing**. Everyday words we might use to describe the black, yellow and green lines might include 'not changing', staying the same. In the Maths world we would describe these lines as being **constant**. The red and brown lines are much steeper than the pink line so we can distinguish between the lines by describing these lines as 'getting higher fairly quickly' or' getting deeper fairly quickly'. In the maths world the words to describe the red and brown lines would be 'sharply increasing'. The blue line shows that the depth of the water is 'going down quickly' or in maths language sharply decreasing.



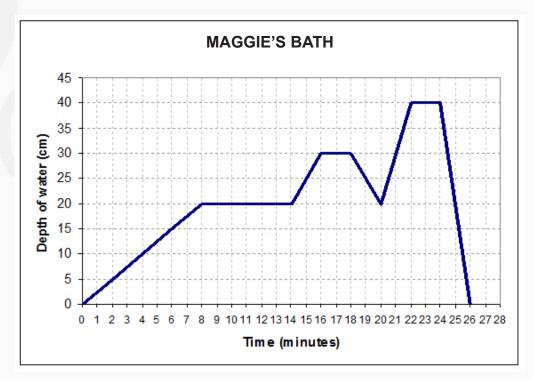
### **Template**



### **Problem Solving and Stories - The Bath Story Work Sheet 2**

### Maggie's Bath

In your own words, tell the story of Maggie's Bath using the information on the line graph below.



Tell the story of Maggie's Bath in your own words:

Re-write the story of Maggie's Bath using words from the "Maths World":

### **Problem Solving and Stories - The Bath Story Work Sheet 3**

Draw a graph of how you would normally take a bath.

ell the st	ory of you	own bath	in (a) you	r own word:	s and (b) u	sing wor	ds from t	he Maths	Wo
Tell the st	ory of you	own bath	in (a) you	r own word	s and (b) u	sing wor	ds from t	he Maths	Wo
Tell the st	ory of you	own bath	in (a) you	r own words	s and (b) u	sing wor	ds from t	he Maths '	Wo
Γell the st	ory of you	own bath	in (a) you	r own word:	s and (b) u	sing wor	ds from t	he Maths	Wo
Fell the st	ory of you	own bath	in (a) you	r own word	s and (b) u	sing wor	ds from t	he Maths	Wo
Tell the st	ory of you	own bath	in (a) you	r own words	s and (b) u	sing wor	ds from t	the Maths	Wo

### **Problem Solving and Stories 2 - Who Lives Where?**

### **Teacher and Tutor Guidelines**

### Aim of the exercise – To work out which family lives in which house.

**Teacher and Tutor/Tutor Guidelines:** Learners work in groups of 2,3 or 6. Make one copy of the problem for each group. Photocopy the templates onto card, cut out clue cards and moveable pieces (the names of the families & clues). The template of the estate is placed in the centre of the table. Each person is given at least one clue that is read out in turn and discussed by the group. Learners shift the moveable pieces as they discuss the implications of each clue and try and find a solution. When the learners think they have the correct solution they read through all the clues once more to check that they are correct.

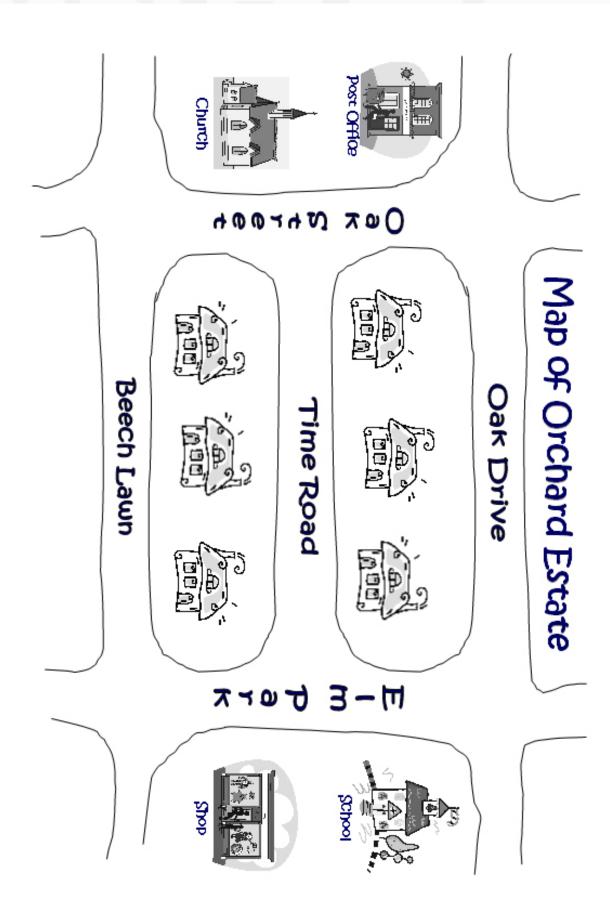
### **Additional Activity:**

The activity can also be used to get learners to reflect on how they work as a group. Did they work effectively? Did one person do all the talking or was there good discussion with negotiation? Why do they think the team worked well or badly? How could they work better next time?

### **Problem Solving and Stories for younger learners – Tallaght Pet Store?**

Give learners the picture of Tallaght Pet Store. Talk about the picture and get the learners to identify what kind of shop it is, what it sells, what kind of animal lives in the different houses outside the shop and so on.

Photocopy the pictures of the dogs onto cards and give one set to each learner or group of learners. The learners must identify which dog they think would live in which kennel. For some learners it might be appropriate to discuss the relative sizes of the dogs.



### Who lives where? Family Cards

O'Sullivan Family

Murphy Family

Todd Family Sweeney Family

Kilbride Family Walsh Family

### Who lives where? Clue Cards

Walking from the church to the shop you pass in front of O'Sullivan's house

Mrs Todd can see
Mrs O'Sullivan
back windows
from her house

The Kilbride family lives furthest way from the post office

The Murphy family lives nearest to the school

The Sweeney children pass in front of Mrs Todds house on their way to school

Mrs Walsh
passes the side of
the Todd house
on the way
to the post office



### **Template**



Developing Maths Eyes A Resource Pack

### **Problem Solving and Stories 3 - Crack the Code!**

### **Teacher and Tutor Guidelines**

People create code to try to hide information. Sport coaches have elaborate methods for giving signs to keep the opposing teams from knowing their plans.

Military leaders also need to keep their plans secret. One of the first military leaders to use coded messages was Julius Caesar. His code involved shifting the postion of the letters in the alphabet. He shifted the letters three spaces forward using a key of +3 as shown below. If you add 3 to the position number of a letter you will get the position of the coded letter. i.e. A is in postion 1. 1 + 3 is postion 4 which is usually the letter D. So when coded, the letter A becomes D.

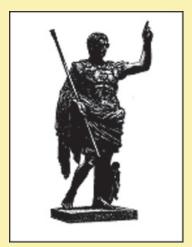


Figure 1 - Julius Caesar

The following quotation from Julius Caesar:

"As a rule men worry more about what they can't see than about what they can."

would be coded as,

"Dv d uxoh phq zruub pruh derxw zkdw wkhb fdq'w vhh wkdq derxw zkdw wkhb fdq."

The number of spaces by which you shift the alphabet is called the KEY.

You can make a codes more secure by changing the numbers of spaces or the direction in which the letters are shifted, giving the secret key only to the person to whom you are sending the secret message.

In Dan Brown's book 'Digital Fortress', the two main characters send notes to each other using a variation of the Caesar code. They use a shift transformation, moving the letters one space forward, (letter + 1). Since both of them know the secret key, they decode each other's messages without anyone else knowing what they are saying.

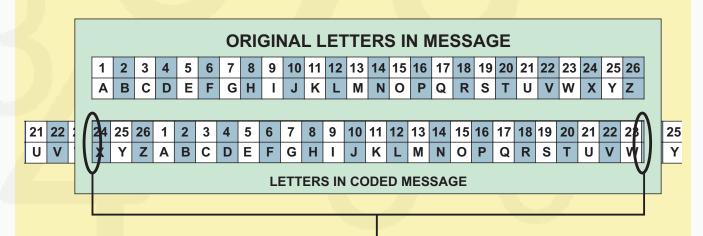
I LOVE YOU is written as J MPWF ZPV.

### **Teacher and Tutor Guidelines**

The main preparatory work is to photocopy the shift cipher device sheet onto card (preferably) and then to:

- 1. Cut out the long Cipher Strip and Cipher Mask
- 2. Cut two slits at the short end of the rectangle in the Cipher Mask so that the long Cipher Strip can slip in and out across the rectangle under the Original Letters in Message part.

To use the cipher device insert long Cipher Strip in through the Cipher Mask upon which the letters of the alphabet have been written. You slide the slip of paper back and forth for different codes. A template has been provided.



Slits in 'Mask' through which the 'Strip' can slide.

### **ANSWER QUESTION 1**

+3: Dwwdfn Yhuflqjhwrula dw gdzq Attack Vercingetorix at dawn

**ANSWER QUESTION 2** 

-10: Dusuiiyjo yi jxu cejxuh ev ydludjyed Necessity is the mother of invention

### **Problem Solving and Stories 3- Crack the Code**

### Worksheet

•••	None of the second seco
You w	will need a $Key = +3$ and the $Key = +1$ to complete the tasks below.
1.	Caesar might have used the shift three spaces forward ( + 3) code to send messages to one of his generals. Translate the message back into its usual form.  Using the Caesar Cypher Strip (Template Attached)
	Dwwdfn Yhuflqjhwrula dw gdzq
2.	Use your Cipher strip to decode this coded message. The message was encrypted (coded with a key of $\Box - 10$ , ten places backwards. (Think of the alphabet as a continues loop).
	Dusuiiyjo yi jxu cejxuh ev ydludjyed
3.	Use the Cipher strip to encode the messages below with a key of $\ \Box$ + 5, (five places forward).
	HAVE INVENTED SELF CLEANING FOOTBALL STRIP.
	MEET ME AT NOON TO DISCUSS THE NEXT PLANNED INVENTION.
4.	Write a note to someone in your group using the shift one forward key code. Trade notes with someone in your group and decode their message.
	Invent your own code using your own secret key.
	Write a note to someone in your group using your code.
	Give them 4 minutes to try and decode it without you giving them the key.
	<ul> <li>After 4 minutes ( if they haven't decoded it ) give them the key and one extra minute to see can they decode it with the key.</li> </ul>
5.	Interview some of your friends. Ask them why encryption techniques are important in today's world. Write down their answers here:

### **Template**

26	N	C	3
25	>	В	2
24	×	A	1
23	>	Z	26
22 23	>	Υ	25
21	)	×	24
9 10 11 12 13 14 15 16 17 18 19 20 21	У X W V U Т S	S T U V W X Y Z A	23
19	S	>	22
18	M N O P O R	U	19 20 21
17	Ø	Τ	20
16	۵	S	19
15	0	8	18
14	Z	Ø	17 18
13	Σ	Ь	16
12		J K L M N O P Q R	10 11 12 13 14 15 16
7		Z	14
10	<b>マ</b>	Σ	13
6	_	L	12
œ	E E	×	11
2 8	G	J	10
9	ш	-	6
2	Е	Н	8
4	Q	9	2
က	ပ	F	9
7	m	Е	2
_	A	Q	4
	ORIGINAL LETTER	CODED	+3

### The Caesar Cipher Strip

**Crack the code** (templates for shift cipher device that will be needed by Learner) 1

**CIPHER MASK** 

## DRICINAL LETTERS IN MESSACE

	8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26	Z
	25	>
	24	×
	23	8
	22	<b>^</b>
	21	n
Ī	20	T
1	19	S
) D	18	R
Ĺ	17	Ø
2 7	16	Ъ
	15	0
	14	Z
-	13	Σ
Ų	12	٦
	11	У
ORIGINAL LE LIERS IN MESSAGE	10	J K L M N O P Q R S T U V W X Y Z
5	6	
	œ	I
	2	9
	9	J
	9	3
	4	Q
	3	၁
	2	B
	1	A

**LETTERS IN CODED MESSAGE** 

**CIPHER STRIP** 

	NOS	HERE
	16	۵
	15	0
	14	Z
	13	Σ
	12	L
	11	X
	10	J
	6	_
	8	I
	7	G
	9	ш
	2	Ε
	4	С
	3	
	2	В
	_	4
	26	7
	25	<b>&gt;</b>
	24	×
	23	<b>8</b>
	22 23	>
	21	n
	20	Н
	19	S
	18	2
5	17	Ø

CUT STRIPS OUT AND JOIN TOGETHER

COL SINIFO COL AND SOIN IOGETHEN	8 19 20 21 22 23 24 25 26 1 2 3 4 5 6 7 8 9 10	U V W X Y Z A B C D E F G H I J	
	1 22 2	<b>&gt;</b>	
	20 2	ST	
	18 19	RS	
<b>→</b>	Join 17	HERE Q	<b>}</b>
			6

### **Section 6 - Maths Diary**

Using a Maths Diary is a good way to reinforce the idea that mathematics forms an integral part of everyday life. The diary can include written, audio, audio-visual or recordings.

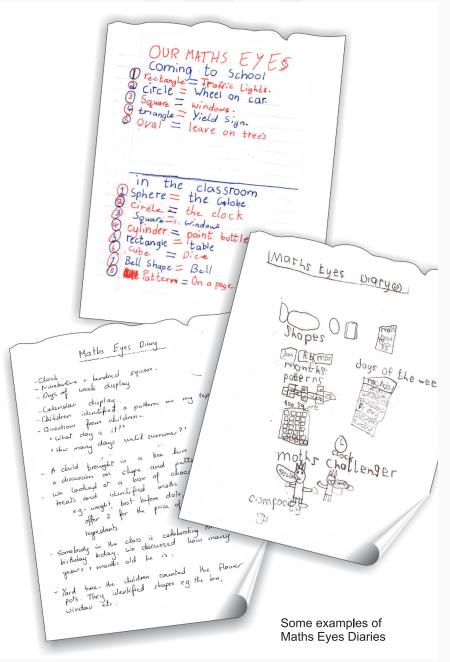
Learners can be asked maintain a Maths Diary over a number of days recording the kinds of maths they have used or seen each day. Learners should be encouraged to bring in "Maths Eyes" pictures that they have taken from their own lives to share with their group.

Using a Maths Diary also highlights especially for those in school, that mathematics is not just something you learn at school but is useful in everyday life.

At the end of each day learners could be asked to record:

- Something they learnt about Maths
- Something they would like to learn about Maths

Maths Diaries/Journals can serve as excellent self-assesment tools as learners record what they have learnt and what they would like to learn more about.



### List of Photographs Available to download from www.haveyougotmathseyes.com

Picture Number	Picture Name	Maths Theme
001	Hearts	<ul><li>Shape and Space – spatial awareness.</li><li>Algebra - extending patterns.</li></ul>
002	Hair Rollers	<ul> <li>Shape and Space - 3 D Shapes</li> <li>Lines</li> </ul>
003	Hopscotch	<ul><li>Algebra</li><li>Shape and Space - 2 D Shapes</li><li>Measures - Length</li></ul>
004	Footpath	<ul> <li>Measures – Area</li> <li>Number – Counting – Operations –         Repeated addition – multiplication</li> <li>Shape and Space</li> </ul>
005	Specials Board	Measures - Money
006	Valentine's Day	❖ Measures - Time
007	Fast Food	<ul><li>Measures – Money</li><li>Shape and Space</li></ul>
008	Windows	<ul><li>Measures – Length</li><li>Shape &amp; Space - 2D -Symmetry</li></ul>
009	Tiled Floor	<ul><li>Shape &amp; Space – Tessellation</li><li>Algebra</li></ul>
010	Arrivals Timetable	<ul> <li>Measures – Time</li> <li>Algebra</li> <li>Sets &amp; Patterns</li> <li>Quantity &amp; Number</li> </ul>
011	Pressure Meters	❖ Quantity & Number
012	Table Plant	❖ Shape and Space

Picture Number	Picture Name	Maths Theme
013	Rainbow Houses	<ul><li>3-D Shapes</li><li>Angles</li></ul>
014	Forks	<ul> <li>Number – Counting – Operations -         Repeated Addition</li> <li>Multiplication</li> </ul>
015	Shirts	<ul> <li>Algebra – Patterns</li> <li>Number – Counting</li> <li>Operations - Multiplication</li> </ul>
016	Speedometer	<ul> <li>Quantity &amp; Number</li> </ul>
017	Approaching Roundabout	❖ Shape
018	Seating Area	<ul><li>Quantity &amp; Number</li><li>Shape</li></ul>
019	Plates	Number - Estimation
020	Side View	<ul><li>❖ Pattern</li><li>❖ Shape</li></ul>
021	Path	❖ Lines & Angles
022	Speed Limit	<ul><li>Measures</li><li>Shapes</li></ul>
023	Woodies	Repeated Addition/Multiplication
024	Roundabout	❖ Shape
025	Pyramid	❖ Shape
026	Clock	<ul><li>Shape</li><li>Measures</li><li>Number</li></ul>
027	Luas Tracks	❖ Parallel Lines

Picture Number	Picture Name	Maths Theme
028	Concrete Ball	❖ Shape
029	Path Decorations	<ul><li>❖ Pattern</li><li>❖ Shape &amp; Space</li></ul>
030	Footsteps	<ul><li>Shape &amp; Space</li><li>Quantity &amp; Number</li><li>Pattern</li></ul>
031	Path Decorations 2	<ul><li>❖ Shape</li><li>❖ Measures</li></ul>
032	Gates	❖ Shape
033	Squares	❖ Shape & Space
034	Estate Agent Sign	<ul> <li>Quantity &amp; Number</li> </ul>
035	Blue Bridge	❖ Shape & Space
036	Glass Roof	❖ Shape
037	Rhubarb Squares	<ul><li>Number – Repeated Addition/Multiplication</li><li>Shape</li></ul>
038	Gingerbread People	❖ Number
039	Spectra Photo	<ul><li>Quantity &amp; Number</li><li>Pattern</li></ul>
040	Photos	❖ Measurement
041	Lift	<ul><li>Shape &amp; Space</li><li>Directions</li></ul>
042	Going Down	<ul><li>Number</li><li>Shape</li></ul>
043	Window Display	<ul><li>❖ Number</li><li>❖ Algebra</li></ul>
044	Butchers	<ul><li>Quantity &amp; Number</li></ul>

Picture Number	Picture Name	Maths Theme
045	Fruit & Veg	❖ Measurement
046	Carrots	<ul><li>Quantity &amp; Number</li><li>Units</li></ul>
047	Green Grocers	❖ Quantity & Number
048	Mandarins	<ul> <li>Measurement Area</li> </ul>
049	Chequered Floor	<ul><li>❖ Algebra - Shape</li><li>- Pattern</li></ul>
050	Steps	<ul> <li>Number</li> <li>Shape &amp; Spaces</li> <li>Angles</li> <li>Measures</li> </ul>
051	Clothes Sales	<ul><li>Number</li><li>Percentage %</li></ul>
052	Budget Travel	<ul><li>Number</li><li>Money</li><li>Measures</li></ul>
053	Holiday Specials	<ul><li>Quantity</li><li>Number</li><li>Percentage %</li></ul>
054	The Square	❖ Shape
055	Tesco	<ul><li>❖ Number</li><li>❖ Percentage %</li></ul>
056	DVD	<ul><li>❖ Number</li><li>❖ Percentage %</li></ul>
057	Flowers	❖ Number
058	Fish	❖ Number
059	Discounts	<ul><li>❖ Number</li><li>❖ Measures</li></ul>
060	Apples	<ul><li>❖ Number</li><li>❖ Measures</li></ul>

Picture Number	Picture Name	Maths Theme
061	Surf Washing Powder	<ul><li>Measures</li><li>Numbers</li></ul>
062	Luas Safety	❖ Shape
063	Dublin Bus	<ul> <li>Quantity &amp; Number</li> </ul>
064	Apple Juice	<ul><li>Quantity &amp; Number</li><li>Pattern</li></ul>
065	Frozen Chicken	<ul> <li>Quantity &amp; Number</li> </ul>
066	Penguin Bars	❖ Number
067	Suitcases	<ul><li>Measures: Money</li><li>Numbers: Precentage Fractions</li></ul>
068	Headstone SB	<ul> <li>Measures</li> <li>Number</li> <li>Examples: True/False – Sometimes True/ Sometimes False</li> </ul>
069	Tyrrell Headstone	❖ Problem Solving
070	Kinsella Headstone	❖ Problem Solving
071	Keogh Headstone	❖ Problem Solving
072	Keating Headstone	❖ Problem Solving
073	Multicoloured Glass	<ul><li>❖ Shape</li><li>❖ Quantity</li></ul>
074	Tallaght Buses	❖ Number
075	Bus Stop	❖ Number
076	Blue Railings	❖ Shape
077	Structure	<b>❖</b> Shape
078	Aerial	❖ Shape
079	Office Railings	❖ Shape
080	Wall	<ul><li>Shape</li><li>Pattern</li></ul>

081	Entrance	<ul><li>❖ Shape</li><li>❖ Pattern</li></ul>
082	Scented Candles	<ul><li>Number</li><li>Algebra</li><li>Pattern</li></ul>
083	Timber	<ul><li>❖ Number</li><li>❖ Measures</li></ul>
084	Wall Tiles	<ul><li>❖ Shape</li><li>❖ Pattern</li></ul>
085	Packet of Batteries	<ul><li>Quantity &amp; Number</li></ul>
086	Eveready Batteries	<ul> <li>Quantity &amp; Number</li> </ul>
087	Garda Station	❖ Shape
088	Pedestrian Crossing	❖ Shape
089	Apartment Block	❖ Shape
090	Directions	<ul><li>Shape &amp; Space</li><li>Directions</li></ul>
091	Watergate Park	❖ Shape

### Also Included:

- Softcopies of the posters are available with the pack along with 10 additional Maths Eyes posters and 15 Maths Problem Picture Posters.
- Interactive Challenge for 'Solve It' Exhibition.

# Notes

# Notes

## Notes



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