ANGRY BI
Mathema
Parabolas and

Vector properties and the birds' frictionless environment help students understand the mathematics behind the game.

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Sometimes the simplest concept can fascinate completely. Who could have guessed that launching a red bird toward a tower of blocks inhabited by egg-stealing green pigs would become as popular a video game as it has? Toddlers to 90-year-olds have become enamored of this simple game rooted in the mathematical concepts of projectile motion. From the moment I first launched a tiny, angry red bird toward those thieving pigs, I became intrigued by the thought of using this game to motivate mathematics students, their teachers, and their future teachers. As a professor of mathematics education and a teacher of high school precalculus, I have developed a way to use the elements of the game Angry Birds® as a platform to engage my students with the concepts of parabolas and vectors.

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SCREEN: COURTESY OF ROVIO; TABLET: THINKSTOCK

The desire to use computers and computer gaming in mathematics education dates to work like that of Feurzeig and Papert (1968, 2011). They developed the computer programing language LOGO specifically for education, resulting in students learning, through a game, to program an on-screen turtle to complete a maze (Papert 1980). These early computer gaming ideas have evolved, incorporating better graphics and placing a greater emphasis on specific mathematical concepts. Kebritchi, Hirumi, and Bai (2010) found that high school students who played an algebra-infused computer game significantly outperformed students who did not. The game-playing students were found to score significantly better on both school benchmark tests and assessments specific to the content of the game. Although research is not consistently positive, much of it supports the conjecture that students learn mathematics from computerized games (Egenfeldt-Nielsen 2007).

Egenfeldt-Nielsen (2006) described one type of computerized game as one that “simulates a part of the world that is simplified and constructed to facilitate working with concrete objects. When interacting with objects in microworlds, we are learning about the objects’ properties, connections, and applications” (p. 198). The game Angry Birds could be categorized as this type of microworld game in which students interact with the properties, connections, and applications of projectile motion. The vector properties of the slingshot and the frictionless environment in which the bird travels coupled with the entertaining story line, exceptional graphics, and catchy sound effects provide a motivational resource for teachers to guide students toward understanding the mathematics behind the game.

Projectile motion is governed by Newton’s three laws of motion:

1. Objects stay at rest or move with a constant velocity unless acted on by some force.
2. The acceleration of a body is directly proportional to, and in the same direction as, the net force acting on the body and is inversely proportional to its mass. Thus, $F = ma$, where F is the net force acting on the object, m is the mass of the object, and a is the acceleration of the object.
3. When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction to that of the first body.

In Angry Birds, the slingshot exerts a force onto the bird at a certain angle that sends the bird into motion until the tower of pigs exerts an equal and opposite force to stop the flight of the bird. Each element of the game has a certain mass and is acted on by the constant downward, or negative, vertical force of gravity. Gravity pulls the bird downward and causes the pigs and towers to crumble, depending on their masses. The game’s programmers did not add horizontal air resistance, which would affect the flight of the bird; therefore, the horizontal velocity of flight remains constant, vertical velocity is governed by gravity, and the resulting trajectory of the bird becomes a parabola.

Parabolas and quadratic functions appear in curricula from upper middle school through calculus. The remainder of this article describes how I have used Angry Birds to engage all my students and focuses on how high school precalculus students were challenged to explore Angry Parabolas, Angry Vectors, and the partnership of these mathematical concepts within Angry Projectile Motion.

ANGRY PARABOLAS

When exploring the Angry Parabola, I begin by launching a bird and asking a simple question such as, “What mathematics do you see?” A bird’s flight path is revealed by the white dots that appear following a launch (see **fig. 1)**. As students share their observations, I urge them to discuss the force, angles, and, more specifically, the parabolic trajectory.

The class discussion then moves toward determining the equation of the trajectory of the bird. Because the game is played in two-dimensional space, a vertical and horizontal coordinate system must be defined before doing so. If we use the vertex form for a quadratic function, $f(x) = a(x - h)^2 + k$, determining the vertex and another point on the parabolic trajectory will allow us to find a value for a.

At this time, **figure 1** remains posted where it is visible to the entire class, and students mark off their coordinates before providing the “angry” coordinate system shown in **figure 2**. I want students to understand the importance of establishing a standard unit of measurement in both dimensions, imperative for any graphical representation of two-dimensional functions. Students begin by suggesting American standard units of measure such as inches. However, given the size of the screen (mobile device, tablet computer, or projection on the whiteboard), they soon realize that other units of measure are preferable. Students then move to ideas such as using the slingshot height as the unit of measure. I then suggest using the diameter of the Angry Bird as the standard unit of measure.

Once a coordinate grid has been established, students approximate the location of the vertex through visual inspection, using the horizontal and vertical distances from the top of the slingshot. Two additional points can be determined by

Fig. 1 The parabolic trajectory of the Angry Bird is highlighted during the launch.

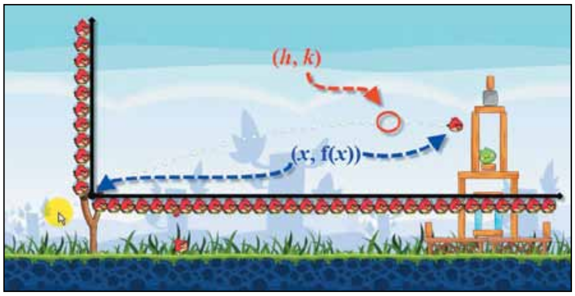

Fig. 2 The diameter of an Angry Bird is the unit of measure on both axes.

approximating the coordinates of a launched bird in midflight or by using the starting position at the top of the slingshot, which would be at the origin. Obviously, a certain amount of guesswork is associated with this activity, and human error plays a role. Because the main idea is for students to explore the concepts of quadratic functions within an engaging connection to this popular game, we accept the human error that necessarily occurs.

I require students to approximate the coordinates of the vertex point to the nearest tenth of an "angry" unit. Using the coordinates (20.2, 4.8) for the vertex and (0, 0) for the point on the curve, we determine the quadratic function for the Angry Parabola in **figure 2**. We first substitute the origin into the vertex form of the quadratic function to obtain $0 = a(0 - 20.2)^2 + 4.8$ and then find that $a \approx -0.0118$. Our equation then becomes $f(x) = -0.0118(x - 20.2)^2 + 4.8$, which can be converted to "standard" form: $f(x) = -0.0118x^2 + 0.47672x$.

Formative and summative assessments are implemented with my students by providing images of Angry Bird trajectories superimposed on the "angry" coordinate plane (see the **appendix**) for students to calculate the parabola, its vertex point, zeros, and other points on the modeled pathway. With my precalculus students, I use this activity early in the course to help students review their knowledge of quadratic functions and how to determine functions through modeling.

ANGRY VECTORS

Vectors are defined by magnitude and direction. Magnitude can be expressed by distance, speed, or translation. The direction of a vector is often denoted as its angular bearing from a constant direction, such as an angle of elevation. The Angry Parabola is a result of two factors: the slingshot force and its angle. The combination of these two factors creates the Angry Vector.

I provide students with **figure 3** and discuss all the Angry Vectors that can be created in the game. The magnitude of the Angry Vector can be defined by the length of the slingshot band as it is stretched, illustrated in **figure 3** by the circles ranging from one-fourth to full power. The direction of the Angry Vector is determined by the angle of rotation of the bird about the top of the slingshot. Using **figure 3**, my class discusses the Angry Vectors created in all four quadrants as well as the probable trajectories following a bird launched with those vectors.

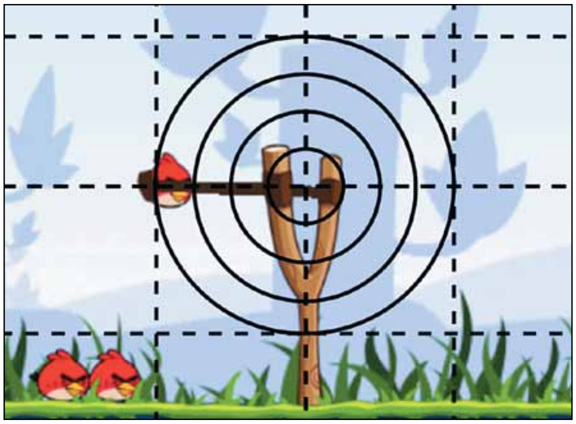

Fig. 3 Circular trigonometry can be used to determine the prelaunch position of the Angry Bird.

Each location of an Angry Bird, as it rotates about the slingshot, is the initial point for an Angry Vector. The coordinates of this point can be determined in either a rectangular or a polar coordinate system. I guide my class, using **figure 3**, through an exploration of rectangular and polar coordinates of various initial points. The rectangular coordinates can be defined by the Angry Bird units in the horizontal and vertical distances from the origin, whereas the polar coordinates can be defined by the radius, or force from the initial point to the origin, and the angle of rotation. The polar coordinates then lend themselves to defining the Angry Vector for each trajectory. Because the pig target in the game is to the right of the Angry Birds, I have my class focus on initial points in quadrant III, which will provide the most successful launches (see **fig. 4,** p. 339).

My class then completes a table of polar and rectangular coordinates for these initial quadrant III points according to their locations when the slingshot is at full power, three-fourths power, one-half power, and one-fourth power (see **table 1**). Students use their knowledge of the unit circle or their knowledge

Table 1 Rectangular and Polar Coordinates of Initial Points at Varying Slingshot Forces							
Slingshot Force	**Angle of Rotation**	**Reference Angle**	**Distance* from Origin**	**Horizontal Distance* from Origin**	**Vertical Distance* from Origin**	**Rectangular Coordinates**	**Polar Coordinates**
1	210°	30°	3	$\frac{3\sqrt{3}}{2}$	$\frac{3}{2}$	$\left(-\frac{3\sqrt{3}}{2},-\frac{3}{2}\right)$	(3, 210°)
	225°	45°	3	$\frac{3\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{2}$	$\left(-\frac{3\sqrt{2}}{2},-\frac{3\sqrt{2}}{2}\right)$	(3, 225°)
	240°	60°	3	$\frac{3}{2}$	$\frac{3\sqrt{3}}{2}$	$\left(-\frac{3}{2},-\frac{3\sqrt{3}}{2}\right)$	(3, 240°)
3/4	210°	30°	$\frac{9}{4}$	$\frac{9\sqrt{3}}{8}$	$\frac{9}{8}$	$\left(-\frac{9\sqrt{3}}{8},-\frac{9}{8}\right)$	$\left(\frac{9}{4}, 210°\right)$
	225°	45°	$\frac{9}{4}$	$\frac{9\sqrt{2}}{8}$	$\frac{9\sqrt{2}}{8}$	$\left(-\frac{9\sqrt{2}}{8},-\frac{9\sqrt{2}}{8}\right)$	$\left(\frac{9}{4}, 225°\right)$
	240°	60°	$\frac{9}{4}$	$\frac{9}{8}$	$\frac{9\sqrt{3}}{8}$	$\left(-\frac{9}{8},-\frac{9\sqrt{3}}{8}\right)$	$\left(\frac{9}{4}, 240°\right)$
1/2	210°	30°	$\frac{3}{2}$	$\frac{3\sqrt{3}}{4}$	$\frac{3}{4}$	$\left(-\frac{3\sqrt{3}}{4},-\frac{3}{4}\right)$	$\left(\frac{3}{2}, 210°\right)$
	225°	45°	$\frac{3}{2}$	$\frac{3\sqrt{2}}{4}$	$\frac{3\sqrt{2}}{4}$	$\left(-\frac{3\sqrt{2}}{4},-\frac{3\sqrt{2}}{4}\right)$	$\left(\frac{3}{2}, 225°\right)$
	240°	60°	$\frac{3}{2}$	$\frac{3}{4}$	$\frac{3\sqrt{3}}{4}$	$\left(-\frac{3}{4},-\frac{3\sqrt{3}}{4}\right)$	$\left(\frac{3}{2}, 240°\right)$
1/4	210°	30°	$\frac{3}{4}$	$\frac{3\sqrt{3}}{8}$	$\frac{3}{8}$	$\left(-\frac{3\sqrt{3}}{8},-\frac{3}{8}\right)$	$\left(\frac{3}{4}, 210°\right)$
	225°	45°	$\frac{3}{4}$	$\frac{3\sqrt{2}}{8}$	$\frac{3\sqrt{2}}{8}$	$\left(-\frac{3\sqrt{2}}{8},-\frac{3\sqrt{2}}{8}\right)$	$\left(\frac{3}{4}, 225°\right)$
	240°	60°	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{3\sqrt{3}}{8}$	$\left(-\frac{3}{8},-\frac{3\sqrt{3}}{8}\right)$	$\left(\frac{3}{4}, 240°\right)$

*All distances are measured in Angry Birds units.

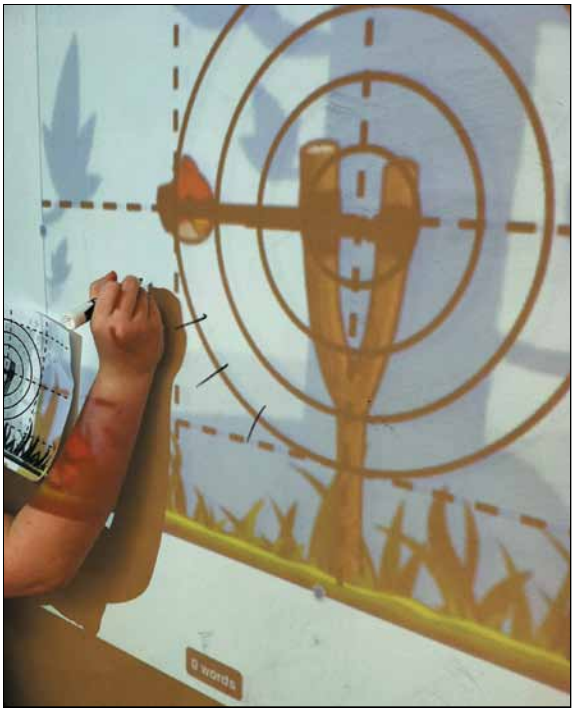

Fig. 4 Students find polar and rectangular coordinates of the Angry Bird prelaunch positions.

of polar equations to complete the table. Once the class has completed the table, discussion related to the magnitude and direction of the Angry Vectors is used to urge students toward the final element of the Angry Bird mathematics: Angry Projectile Motion.

ANGRY PROJECTILE MOTION

The magnitude of the Angry Vector within the kinematic components of projectile motion is the initial velocity of the Angry Bird, V_a, resulting from the slingshot's force. The direction of the Angry Vector is determined by the standard position angle, θ (see **fig. 5**). As discussed earlier, the Angry Bird travels in a parabolic trajectory because the only force programmed to act on the bird, after it is launched, is that of the downward pull of gravity. The Angry Bird's position (the ordered pair (x, y)) after a given amount of time is governed by two kinematic equations:

$$x = x_0 + V_{0_x} t$$

$$y = y_0 + V_{0_y} t - \frac{1}{2} g t^2$$

In these equations, V_{0_x} and V_{0_y} are the initial velocities in the horizontal and vertical directions, respectively; g is the downward force of gravity ($9.81\ \text{m/s}^2$); t is time; and x_0 and y_0 are the initial rectangular coordinates of the object being projected. Class discussion then moves toward the discovery of the Angry Vector's initial velocity, V_a, and direction, θ (see **fig. 5**).

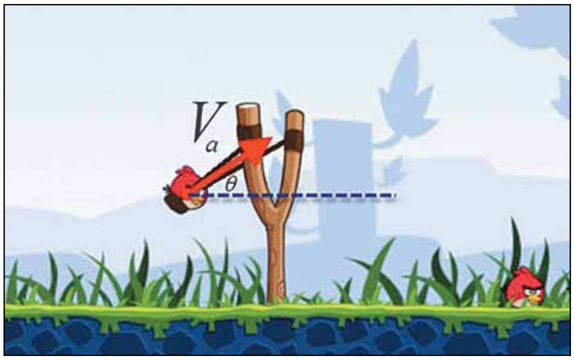

Fig. 5 Angry Vectors are defined by the initial velocity and angle of elevation of the launched Angry Bird.

Right-triangle trigonometry yields a relationship between the initial velocity of the Angry Vector and the measurements of V_{0_x}, V_{0_y} and θ:

$$\sin\theta = \frac{V_{0_y}}{V_a};\ \cos\theta = \frac{V_{0_x}}{V_a};\ \text{and} \tan\theta = \frac{V_{0_y}}{V_{0_x}}.$$

The class can then use the standard quadratic equation form of $y = ax^2 + bx + c$ to solve for V_a and θ. The following verifies what a, b, and c equal in terms of V_{0_x}, V_{0_y}, and g:

$$x = x_0 + V_{0_x} t \rightarrow t = \frac{x - x_0}{V_{0_x}}$$

We use this value for t and substitute as follows:

$$\begin{aligned}
y &= y_0 + V_{0_y} t - \frac{1}{2} g t^2 \\
&= y_0 + V_{0_y} \cdot \frac{x - x_0}{V_{0_x}} - \frac{1}{2} g \left(\frac{x - x_0}{V_{0_x}} \right)^2 \\
&= y_0 + \frac{V_{0_y}}{V_{0_x}} x - \frac{V_{0_y}}{V_{0_x}} x_0 - \frac{g\left(x^2 - 2x_0 x + x_0^{\ 2}\right)}{2\left(V_{0_x}\right)^2} \\
&= y_0 + \frac{V_{0_y}}{V_{0_x}} x - \frac{V_{0_y}}{V_{0_x}} x_0 - \frac{g}{2\left(V_{0_x}\right)^2} x^2 + \frac{g x_0}{\left(V_{0_x}\right)^2} x - \frac{g x_0^{\ 2}}{2\left(V_{0_x}\right)^2} \\
&= \left(-\frac{g}{2\left(V_{0_x}\right)^2} \right) x^2 + \left(\frac{V_{0_y}}{V_{0_x}} + \frac{g x_0}{\left(V_{0_x}\right)^2} \right) x \\
&\quad + \left(-\frac{V_{0_y}}{V_{0_x}} x_0 - \frac{g x_0^{\ 2}}{2\left(V_{0_x}\right)^2} + y_0 \right)
\end{aligned}$$

Following this verification, the class can take an Angry Parabola in standard form and determine V_a and θ. We use the parabola $y = -0.0118x^2 + 0.47672x$ (whose equation we determined earlier) and $g = 9.81$ "angry" units/s^2, and begin by equating the two expressions for a:

$$-0.0118=-\frac{9.81}{2\left(V_{0_x}\right)^2}\rightarrow$$
$$V_{0_x}=\sqrt{\frac{9.81}{2(0.0118)}}\approx 20.3882$$

We then do the same for our representations of *b*:

$$0.47672=\frac{V_{0_y}}{V_{0_x}}\rightarrow$$
$$V_{0_y}=0.47672\bullet 20.3882\approx 9.7195$$

Then $\tan\theta = V_{0_y}/V_{0_x} \rightarrow \tan\theta = 0.47672 \rightarrow \theta \approx 25.4881°$. We have $\sin\theta = V_{0_y}/V_a \rightarrow V_a = V_{0_y}/\sin\theta \rightarrow V_a \approx 9.6885/\sin 25.1881° \approx 9.6885/.4303 \approx 22.5864$. Thus, the initial velocity of the Angry Bird is nearly 23 "angry" units per second at an angle of slightly more than 25 degrees.

DISCUSSION

All my students have become more engaged and intrinsically motivated to learn the mathematical concepts emphasized in these lessons using Angry Parabolas, Angry Vectors, and Angry Projectile Motion. Middle school students seemed to clamor to measure angles and distances so that they could get a chance to launch an Angry Bird toward a tower of pigs. Algebra students, despite moaning that I have ruined their favorite game, were able to understand how scientific elements of vectors directly connected to algebraic modeling and prediction through parabolas. Teachers embraced the possibilities of using these concepts in their classrooms, and many contacted me with their adaptations.

Teachers of mathematics have an important role in developing student thinking. The concepts taught throughout mathematical coursework help build critical thinkers who will become more productive employees and leaders in their future careers. Too often, students fail to see a connection between the mathematics that they are learning and its application in the real world. Using Angry Birds can help students explore mathematical concepts in ways that have direct appeal.

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APPENDIX

Below are two images suitable for student assessment. For additional similar images, go to **www.nctm.org/mt046**.